

Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. (20 points) Find the derivative for the following functions. Do not simplify.

(a) $f(x) = \frac{6x}{x^2 + 3} + e^{2x} \arctan(x)$.

(b) $g(x) = \ln(x^2 + 1) - \int_x^1 \ln(t) dt$

2. (16 points) Use the limit definition of derivative to find the derivative of $f(x) = \frac{1}{x}$.

3. (36 points) Evaluate the following integrals.

(a) $I = \int_{-3}^3 x \cos^2(x) + |x| dx$

(b) $I = \int_0^1 3x\sqrt{2-x^2} dx$

(c) $I = \int_0^\pi (2x+1) \cos(x) dx$

4. (16 points) (A) Find the equation of the tangent line to the curve $x^3 + y^2 = x + y^3$ at the point $(1, 1)$.

(B) Use your work from part (A) to write down the linearization of y near $(1, 1)$ and then use it to estimate y when $x = 0.9$.

5. (16 points) Shade the region that is bounded above by $y = 8 - x^2$ and below by $y = x^2$, and then use integration to compute the corresponding area.

6. (16 points) A poster of area 200 cm^2 has blank margins of width 2 cm on the top and bottom and 1 cm on the sides. Find the dimensions of the poster that will maximize the printed area. Use a derivative test to verify your answer is a maximum.

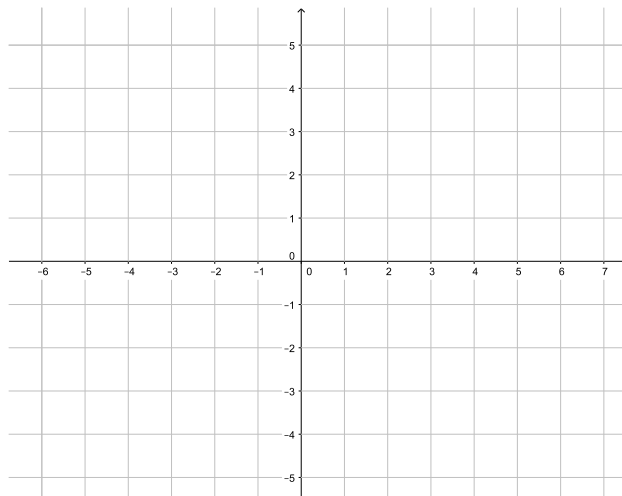
7. (16 points) At time $t = 0$ second, a particle is traveling at 4 m/sec along a line with deceleration $a(t) = -4 \sin(t)$. How far does the particle travel during its deceleration before stopping?

8. (12 points) The radius of a circular oil puddle expands at a rate of 10 meters per minute. How fast is the area of the oil puddle increasing when the radius is 5 meters? Defend your answer with organized work.

9. (12 points) Calculate $f''\left(\frac{\pi}{12}\right)$ if $f(x) = \cos^2(2x)$. Defend your answer with organized work.

10. (12 points) Sketch a possible graph for $y = f(x)$ if

$$f(-x) = f(x), \quad f'(x) > 0 \text{ if } x > 0 \quad f''(2) = 0 \quad \text{and} \quad \lim_{x \rightarrow \infty} f(x) = 3.$$



11. (8 points) Find a definite integral equal to $\lim_{N \rightarrow \infty} \frac{4}{N} \sum_{j=1}^N \ln \left(3 + \frac{4j}{N} \right)$

12. (20 points) Find the limit or infinite limit; otherwise write “DNE” for “does not exist.” Defend your answers and use proper notation.

(a) $\lim_{x \rightarrow 0^-} \frac{3x^2 + x^{-2}}{x^2 - x^{-1}}$

(b) $\lim_{t \rightarrow 0} \frac{\sin(t) \cos(t)}{t(4 - t)}$

(c) $\lim_{x \rightarrow \infty} x e^{-x}$