25 points | Math 150 | Homework 1 | Secions 2.1-2.5 | 23 problems

Homework is graded for completeness and presentation; only the CAS problem is graded for correctness. Please make a margin on the left and put your circled problem numbers in order to the left of the margin. Leave space between each problem. Leave a margin on top so that when the papers are stapled the problem number in the upper left corner is visible, or put the problem number in the center of the page. Provide context for each problem. One-word answers rarely earn credit. Each problem is worth at least one point, even if there are 100 problems and only 25 points to award. The answer for the CAS problem may be placed before problem 1 instead of last if you wish. Please see the calendar for the due date.

1. Sketch the graph of $f(x) = x(1-x)$ over $0 \leq x \leq 2$. Find the following.
   (a) The average rate of change over the interval $[0, 1]$.
   (b) The average rate of change over the interval $[0, 2]$
   (c) The rate of change at $x = 0.5$.
   Hint: rate of change at one point means instantaneous rate of change.
   (d) The values of x at which the rate of change is negative over the interval $[0, 2]$.

2. Use the tabular method to estimate $\lim_{x\to 0}|x|^x$. Then check your limit by using a graph generated by technology (for instance, Wolfram Alpha free website will easily plot $y = |x|^x$.)

3. Determine the one-sided limits and limits of the function $f(x)$ at $x = -4$, $x = 1$, and $x = 4$. Write DNE if the limits are neither real numbers nor infinite.

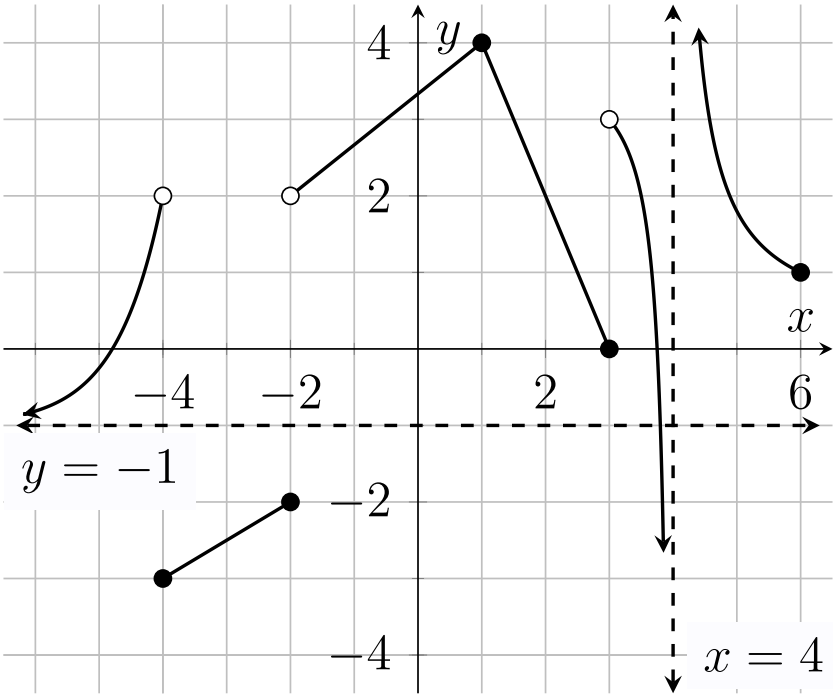

Figure 1: $y = f(x)$

4. Sketch the graph of a function $g(x)$ that has two positive infinite one-sided limits at $x = 3$. What then is $\lim_{x\to 3} g(x)$?

5. Does $\lim_{x\to 0} \cos\left(\frac{\pi}{x}\right)$ exist? Defend your answer using the tabular method. Hint: try two different sequences that get close to zero but have different outputs such as $a_n = \frac{1}{2n}$ and $b_n = \frac{1}{2n+1}$.

6. Determine the points of discontinuity for $h(x) = \frac{\cos(x^2)}{x^2-1}$.

7. Sketch a graph of $p(x) = \begin{cases} \frac{x-2}{|x-2|} & x \neq 2 \\ -1 & x = 2 \end{cases}$ and then determine the points of discontinuity along with the one-sided limits at each point of discontinuity.

8. Find the values of the constants a and b that make $k(x) = \begin{cases} x^{-1} & \text{if } x < -1 \\ ax+b & \text{if } -1 \leq x \leq 2 \\ x^2 & \text{if } x > 2 \end{cases}$ continuous for all real inputs.

9. Sketch a graph of a function $f(x)$ with domain $[0,5]$ that is left-continuous but not continuous at $x = 2$ and right-continuous but not continuous at $x = 3$. "left-continuous" means $\lim_{x\to a^-} f(x) = f(a)$; a similar definition holds for "right-continuous."

10. Sketch the graph of two functions $f(x)$ and $g(x)$ that are discontinuous at a common input c but have a sum $f(x) + g(x)$ that is continuous at c.

11. Find all values of the constant k such that $\lim_{x\to k} \frac{x^2-x-6}{x-k}$ exists. Defend your answer.

Find each limit. Defend your answers using algebra. The tabular method or graphs can be used to check your answers, but not as a defense.

12. $\lim_{t\to 25} \frac{2\sqrt{t}-6t}{(t-15)^2}$

13. $\lim_{\theta\to\pi/12} \tan(4\theta)$

14. $\lim_{y\to 2} \frac{3y^2-4y-4}{3y^2-12}$

15. $\lim_{m\to -1} \frac{m+1}{(2m^2+7m+5)^2}$

16. $\lim_{x\to 4^-} \sin^{-1}\left(\frac{x}{4}\right)$

17. $\lim_{h\to 0} \frac{\frac{1}{(h+3)^2} - \frac{1}{9}}{h}$

18. $\lim_{x\to 0} \frac{\cot(x)}{\csc(x)}$

19. $\lim_{y\to 1} \frac{y^2-5y+4}{y^3-1}$

20. $\lim_{x\to 5} \frac{x}{(x-5)^2}$

21. $\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$

22. $\lim_{\theta\to\pi/4} \left(\frac{1}{\tan(\theta)-1} - \frac{1}{\tan^2(\theta)-1}\right)$

23. CAS problem (3 points): use a CAS device to find the following limits. Submit a printed copy of the device's solution and your corresponding commands.

(a) $\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$

(b) $\lim_{x\to 0^+} \frac{2x^2+4x}{x^3-x^2}$

Brief answers

1. (a) 0, (b) -1, (c) 0, (d) $x \in (0.5, 2)$.
2. 1
3. $\lim_{x\to -4^-} f(x) = 2$, $\lim_{x\to -4^+} f(x) = -3$, $\lim_{x\to -4} f(x)$ DNE, $\lim_{x\to 1^-} f(x) = 4$, $\lim_{x\to 1^+} f(x) = 4$, $\lim_{x\to 1} f(x) = 4$, $\lim_{x\to 4^-} f(x) = -\infty$, $\lim_{x\to 4^+} f(x) = \infty$, $\lim_{x\to 4} f(x)$ DNE.
4. ∞
5. No, it does not exist.
6. ± 1
7. Discontinuous at $x = 2$; $\lim_{x\to 2^-} p(x) = -1$ and $\lim_{x\to 2^+} p(x) = 1$.
8. $a = \dfrac{5}{3}$ and $b = \dfrac{2}{3}$.
9. There are many possible correct solutions; a classic one is a "box" function.
10. There are many possible correct solutions; a classic one uses $\pm$ Heaviside function.
11. $k = 3$ or $k = -2$.
12. -1.4
13. $\sqrt{3}$
14. $\dfrac{2}{3}$
15. DNE
16. $\dfrac{\pi}{2}$
17. $\dfrac{-2}{27}$
18. 1
19. -1
20. ∞
21. $\dfrac{1}{2\sqrt{a}}$
22. DNE