25 points Math 150 Homework 2 Secions 2.6-2.9 23 problems

1. Evaluate $\lim_{t\to 2}(t^2-4)\cos\left(\frac{1}{t-2}\right)+3t$ using the squeeze theorem.

2. Evaluate $\lim_{\theta\to 0^+}\sqrt{\theta}e^{\sin(\theta^{-1})}$ using the squeeze theorem.

Evaluate the following limits and defend your answers with appropriate work.

3. $\lim_{x\to 0}\frac{\sin(4x)}{x}$

4. $\lim_{x\to -\infty}\frac{3x^3-10}{x+4}$

5. $\lim_{x\to 0}\frac{\sin(4x)}{\sin(8x)}$

6. $\lim_{t\to\infty}\frac{\sqrt{9t^4+3t+2}}{4t^3+1}$

7. $\lim_{\theta\to 0}\frac{\sec(\theta)-1}{\theta}$

8. $\lim_{y\to\infty}\frac{y^{4/3}-9y^{1/3}}{(27y^4-8)^{1/3}}$

9. $\lim_{k\to\pi}\frac{\sin(9k)}{k-\pi}$

10. $\lim_{x\to\infty}(\ln(3x+1)-\ln(2x-1))$

11. Sketch and use a graph to determine $\lim_{x\to\infty}\tan^{-1}(x)$.

12. Find the horizontal asymptotes for $f(x)=\frac{2x^2-3x}{8x^2+4}$.

13. Find the horizontal asymptotes for $g(t)=\frac{e^t}{1+e^t}$.

In the following problems, "IVT" means "Intermediate Value Theorem."

14. Use the IVT to show that $f(x)=x^2\tan(x)$ equals $1/2$ for some value x in the interval $[0,\pi/4]$.

15. Use the IVT to show that $\cos(x)=x$ has a solution.

16. Use the IVT to show that $e^x+\ln(x)=0$ has a solution.

17. Let $f(x)=x$ and $g(x)$ be any continuous real-valued function with range contained in the interval $[0,1]$. Use the IVT to show there exists a number c in the interval $[0,1]$ so that $f(c)=g(c)$.

18. Let $f(x)=2x-5$. For an arbitrary $\epsilon>0$, find a corresponding δ in terms of ϵ so that $|f(x)-1|<\epsilon$ whenever $|x-3|<\delta$. Your work should defend your answer. Once finding that such a δ exists, you have proven what limit exists?

19. Let $f(x)=2x-5$. Find all possible $\delta>0$ so that $x\in[3-\delta,3+\delta]$ implies $f(x)\in[.9,1.1]$.

20. Let $f(x) = 8x + 1$. For an arbitrary $\epsilon > 0$, find a corresponding δ in terms of ϵ so that $|f(x) + 7| < \epsilon$ whenever $|x + 1| < \delta$. Your work should defend your answer. Once finding that such a δ exists, you have proven what limit exists?

21. Let $f(x) = \dfrac{1}{x}$. For an arbitrary $\epsilon > 0$, find a corresponding δ in terms of ϵ so that $|f(x) - 0.25| < \epsilon$ whenever $|x - 4| < \delta$. Your work should defend your answer. Once finding that such a δ exists, you have proven what limit exists?

22. Let $f(x) = \dfrac{1}{x}$. Find all possible $\delta > 0$ so that $x \in [4 - \delta, 4 + \delta]$ implies $f(x) \in [0.24, 0.26]$.

23. CAS problem (3 points): use a CAS device to find the following limits. Submit a printed copy of the device's solution and your corresponding commands.

    (a) $\lim_{x\to\infty} e^{-x} \sin(x)$ (b) $\lim_{x\to\infty} \dfrac{\sqrt{x^{1/3} - x^{1/6} + 3}}{6x^{1/6} + 1}$

Brief answers

1. 6
2. 0
3. 4
4. ∞
5. 1/2
6. 0
7. 0
8. 1/3
9. -9
10. $\ln(3/2)$
11. $\pi/2$
12. $y = 1/4$ for left and right.
13. $y = 1$ on right, $y = 0$ on left.
14. Hint: $\pi^2/16 > 1/2$.
15. Hint: let $f(x) = \cos(x) - x$.
16. Hint: try $x = 1$ and $x = e^{-10}$.
17. Hint: set $h(x) = g(x) - x$ and discuss why $h(0)$ and $h(1)$ must have opposite signs if neither is zero.
18. $\delta = \dfrac{\epsilon}{2}$; $\lim_{x\to 3} f(x) = 1$.
19. $0 < \delta \le 0.05$ Notice how 0.05 is $\epsilon/2$ from the previous problem.
20. $\delta = \dfrac{\epsilon}{8}$; $\lim_{x\to -1} f(x) = -7$.
21. $\delta = \min\{1, 12\epsilon\}$; $\lim_{x\to 4} f(x) = .25$. Note: there are many possible answers for δ depending on the first restriction (1 in this answer.)
22. $0 < \delta \le \dfrac{2}{13}$ Notice how $\dfrac{2}{13} \approx 0.15$ is larger than $12\epsilon = 0.12$ from the previous problem. This difference is due to the restriction of 1 used defining δ.