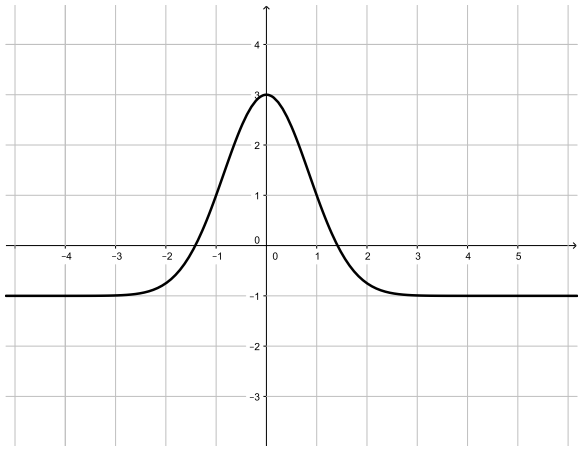
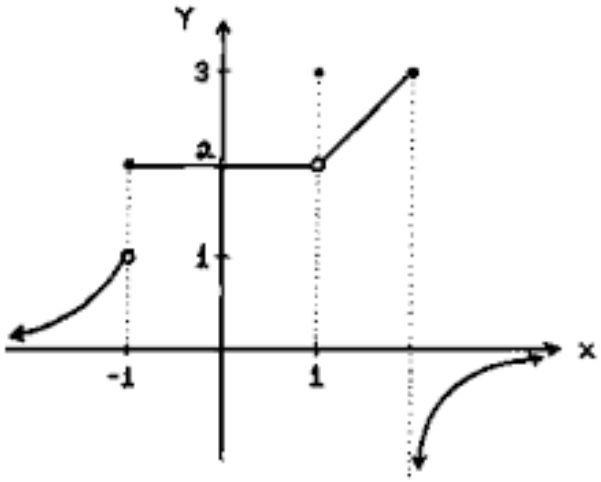
25 points | Math 150 | Homework 3 | Secions 3.1-3.5 | 23 problems

For each graph of a function,

1. The graph of $y = f(x)$ is shown below. Estimate the following: $f'(-4)$, $f'(-1)$, $f'(0)$, $f'(1)$, and $f'(4)$. Use these values to sketch a graph of $y = f'(x)$.



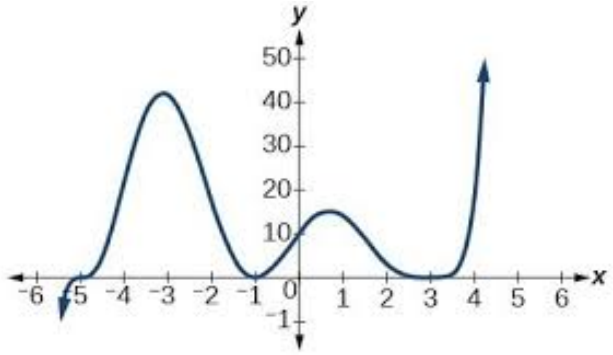
2. The graph of $y = g(x)$ is shown below. Estimate the following: $g'(-2)$, $g'(-1)$, $g'(0)$, $g'(1.5)$, and $g'(3)$. Use these values to sketch a graph of $y = f'(x)$.



3. Use the limit definition to find $h'(-2)$ if $h(x) = \dfrac{1}{x+3}$.

4. If $f'(a) = \lim\limits_{h\to 0} \dfrac{5^{2+h} - 25}{h}$, then what are $f(a+h)$, $f(a)$, $f(x)$ and a?

5. The graph of $p(x)$ is shown below. Estimate the x-axis intervals for which $\dfrac{dp}{dx}$ is negative. Estimate the value of $x \in [-5, 3]$ that makes $p'(x)$ largest.



6. Compute $f'(4)$ if $f(x) = 5x - 32\sqrt{x} + 3x^{3/2}$.

7. Calculate $\dfrac{d}{dt}(25t^{0.4} - 8e^t)$.

8. Calculate $f'(p)$ if $f(p) = \sqrt[4]{p} + \sqrt[3]{p}$.

9. Compute $\dfrac{d}{dz}(cz^4 - 5c + ce^z)$ if c is a constant.

10. Let $g(x) = |x^2 - 1|$. Find the points c for which $g'(c)$ is not defined. Defend your answer.

11. Find the equation of the tangent line to the graph of $h(x) = x^{1/3}$ at $x = 8$.

12. Calculate $\left.\dfrac{df}{dx}\right|_{x=1}$ if $f(x) = (4e^x - x^2)(x^3 + 1)$.

13. Calculate $\left.\dfrac{dy}{dx}\right|_{x=2}$ if $y = \dfrac{x^4 - 4}{x^2 - 5}$.

14. Calculate $f'(x)$ if $f(x) = (x-1)(x-2)(x-3) + 3^2 \cdot 5^{1/3}$.

15. Calculate $g'(x)$ if $g(x) = \dfrac{x + e^x}{x + 1}$.

16. Calculate $(hk)'(4) - \left(\dfrac{k}{h}\right)'(4)$ if $h(4) = 2$, $h'(4) = -2$, $k(4) = 5$, and $k'(4) = -1$.

17. Find $a > 0$ so that the tangent line to the graph of $f(x) = x^2 e^{-x}$ at $x = a$ passes through the origin.

18. Find y'' and y''' if $y = 125t^{4/5} + 27t^{2/3} + t^5 e^t$.

19. Calculate $\left.\dfrac{d^3 f}{dx^3}\right|_{x=-3}$ if $f(x) = 4e^x - x^3$.

20. Find a general formula for $f^{(n)}(x)$ if $f(x) = x^n$.

21. The **height** of a helicopter t minutes after take off for a twenty minute ride is $s(t) = 60t^2 - 2t^3$. Find the velocity and the acceleration at time t. At what values of t is the acceleration negative? What does that tell us about the helicopter's velocity?

22. The dollar cost of producing x bagels is $C(x) = 300 + 0.25x - 0.5\left(\frac{x}{1000}\right)^3$. Use a derivative to estimate the cost of producing the 2001^{st} bagel, and compare that to the actual cost of producing that same bagel.

23. CAS problem (3 points): use a CAS device to solve the following problems. Submit a printed copy of the device's solution and your corresponding commands.

    a) Differentiate $f(x) = \sin(\sinh(x))$ and then approximate $f'(\ln(2))$ to four decimal places. Some CAS's, such as Matlab, use log for the natural log instead of ln.

    b) Differentiate $g(x) = e^{-3x}\cos(x^2)$ and then approximate $g'(\sqrt{\pi/3})$ to four decimal places. Some CAS's use $\exp(x)$ instead of e^x.

Brief answers

1. 0, 2, 0, -2, 0
2. 0.3, DNE, 0, 1, 0.3
3. -1
4. 5^{2+h}, 5^2, 5^x, and 2.
5. $(-3,-1)$ and $(.9,3)$; $x \approx -4$.
6. 6
7. $10t^{-0.6} - 8e^t$
8. $\frac{1}{4}p^{-3/4} + \frac{1}{3}p^{-2/3}$
9. $4cz^3 + ce^z$
10. 1 and -1.
11. $y = \frac{x}{12} + \frac{4}{3}$
12. $20e - 7$
13. -80
14. $3x^2 - 12x + 11$
15. $\frac{xe^x + 1}{(x+1)^2}$
16. -14
17. $a = 1$
18. $y'' = -20t^{-6/5} - 6t^{-4/3} + e^t(t^5 + 10t^4 + 20t^3)$ and $y''' = 24t^{-11/5} + 8t^{-7/3} + e^t(t^5 + 15t^4 + 60t^3 + 60t^2)$.
19. $4e^{-3} - 6$
20. $n!$ or less concisely $n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1$.
21. $v(t) = s'(t) = 120t - 6t^2$, $a(t) = s''(t) = 120 - 12t$. $a(t) < 0$ if $t \in (10, 20]$. Helicopter's vertical **velocity** is decreasing over that interval.
22. Estimated cost $\approx C'(2000) = 0.244$ dollars. Actual cost $= C(2001) - C(2000) \approx 0.2439969995$ dollars.