25 points Math 150 Homework 4 Secions 3.6-3.10 23 problems

1. Differentiate $q(x) = \dfrac{1+\tan(x)}{1-\tan(x)}$

2. Differentiate $g(\phi) = e^{\phi}(5\sin(\phi) - 4\cos(\phi))$

3. Differentiate $R(y) = \dfrac{3\cos(y)-4}{\sin(y)}$

4. Differentiate $y = \dfrac{\sin(x)}{\sin(2x)}$

5. Calculate $f''(\theta)$ if $f(\theta) = \theta\sin(\theta)$.

6. Differentiate $g(x) = \sec\left(\dfrac{x}{x-1}\right)$

7. Differentiate $y = e^{x+x^{-1}}$

8. Differentiate $h(x) = (x^6 - x^3 - 1)^{2/3}$

9. Differentiate $y = (\cos(6x) + \sin(x^2))^{1/2}$

10. Compute $\dfrac{d^3}{dx^3}(9-x)^8$.

11. Differentiate $y = 2^{x^3} - \ln\left[(x+1)(2x+3)\right]$.

12. Find the values of x between 0 and 2π where the tangent line to $y = \sin(x)\cos(x)$ is horizontal.

13. Find the equation for the tangent line to the graph of $s(t) = \ln(8-4t)$ at the point $(1, s(1))$.

14. Find the derivative of $f(x) = \sinh(\ln(x)) - x^{\cos(x)}$.

15. Differentiate $y = \sqrt{1-t^2} + \sin^{-1}(t) - \tan^{-1}(2t)$.

16. **Prove** that for $x > 1$ the derivative of $\cosh^{-1}(x)$ is $\dfrac{1}{\sqrt{x^2-1}}$.

17. Find the equation of the tangent line to the graph of $y = f(x)$ if $x^2 + \sin(y) = xy^2 + 1$ at the point $(1, 0)$.

18. Find the equation of the tangent line to the graph of $y = f(x)$ if $e^{2x-y} = \dfrac{x^2}{y}$ at the point $(2, 4)$.

19. The radius of a circular oil slick increases at a rate of 2 meters/minute. How fast is the area of the oil slick changing when the radius is 3 meters?

20. The bottom of a five meter ladder slides away from a wall while the top of the ladder remains on the wall. Let $h(t)$ represent the height in meters of the top of the ladder on the wall at time t in seconds. If $h(0) = 4$ and $h'(t) = -1$, calculate the distance the bottom of the ladder is from the wall and its rate of change at $t = 1$.

21. A two meter tall man walks away from a five meter lamppost at a rate of one meter per second. Find the constant rate at which his shadow changes in length.

22. Suppose a new anti-gravity device allows a rocket to obtain a constant vertical speed of 1200 km/hr instantaneously. An observer 20 km away from the launching pad observes the rocket with a telescope. Assuming the ground between the observer and the launching pad is flat and perpendicular to the path of the rocket, find the rate at which the angle between the telescope and the ground is increasing three minutes after lift-off.

23. CAS problem (3 points): use a CAS device to solve the following problem. Submit a printed copy of the device's solution and your corresponding commands.

    $s(t) = 2e^{-1.5t}\cos(2\pi t)$ is the number of centimeters displacement of a shock absorber t seconds after the car hits a bump. Find the instantaneous velocity after t seconds and graph both displacement and velocity functions for $0 \le t \le 2$ on the same axis.

Brief answers

1. $\dfrac{2\sec^2(x)}{(1-\tan(x))^2}$
2. $e^{\phi}(9\sin(\phi)+\cos(\phi))$
3. $\dfrac{4\cos(y)-3}{\sin^2(y)}$
4. $\dfrac{\sec(x)\tan(x)}{2}$
5. $2\cos(\theta)-\theta\sin(\theta)$
6. $\dfrac{-1}{(x-1)^2}\sec\left(\dfrac{x}{x-1}\right)\tan\left(\dfrac{x}{x-1}\right)$
7. $e^{x+x^{-1}}(1-x^{-2})$
8. $\dfrac{4x^5-2x^2}{(x^6-x^3-1)^{1/3}}$
9. $\dfrac{2x\cos(x^2)-6\sin(6x)}{2\sqrt{\cos(6x)+\sin(x^2)}}$
10. $-336(9-x)^5$
11. $2^{x^3}(\ln(8)x^2)-\dfrac{1}{x+1}-\dfrac{2}{2x+3}$
12. $x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$
13. $y = -x+\ln(4)+1$
14. $\dfrac{\cosh(\ln(x))}{x} - x^{\cos(x)}(\cos(x)/x-\ln(x)\sin(x))$
15. $\dfrac{\sqrt{1-t}}{\sqrt{1+t}}-\dfrac{2}{1+4t^2}$
16. Hint: $\cosh^2-\sinh^2(x)=1$.
17. $y=-2x+2$
18. $y=\dfrac{4}{3}x+\dfrac{4}{3}$
19. 12π meter^2/minute
20. 4 meters; 0.75 meters/second.
21. 2/3 meters/second
22. 6 radian/hour = 0.1 radian/minute