1. Use a differential to estimate $\Delta f = f(3.02) - f(3)$ if $f(x) = \tan\left(\frac{\pi x}{3}\right)$.

2. Estimate $\sqrt[3]{27.2} - \sqrt[3]{27}$ using a differential.

3. Use a differential to estimate Δf for $f(x) = \dfrac{1}{1+x^2}$ at $x = 3$ if $\Delta x = 0.5$.

4. Find the linearization of $T(\theta) = \sin^2(\theta)$ at $\theta = \pi/4$. Note: this is also called the "first degree Taylor Polynomial" of $T(\theta)$ at $\theta = \pi/4$.

5. Find the linearization of $g(x) = e^x \ln(x)$ at $x = 1$.

6. Find the linearization of $h(x) = e^x \cos(x)$ at $x = 0$.

7. The edge of a cube was measured to be 30 ± 0.1 cm. Use differentials to estimate the maximum percentage error for calculating the surface area of the cube using 30 cm for the edge length. Recall percentage error is $\dfrac{\Delta A}{A} \cdot 100$.

8. The radius of a spherical ball is measured to be $r = 25 \pm 0.5$ cm. Use differentials to estimate the maximum relative error using $r = 25$ cm to calculate the volume of the ball. Recall relative error is $\dfrac{\Delta V}{V}$.

9. Find a point c satisfying the conclusion of the MVT (Mean Value Theorem) for $y = x^{-1}$ on the interval $[2, 8]$. Sketch the corresponding picture that shows a tangent line parallel to a secant line.

10. Find a point c satisfying the conclusion of the MVT for $y = e^x$ on the interval $[0, 1]$. Sketch the corresponding picture that shows a tangent line parallel to a secant line.

11. Find all critical points for $f(x) = x - 4\sqrt{x+1}$.

12. Find the maximum and minimum of $y = \theta - 2\sin(\theta)$ on the interval $[0, 2\pi]$.

13. Find the maximum and minimum of $g(t) = 3e^t - e^{2t}$ on the interval $[0, 10]$.

14. Find and classify the critical points of $g(y) = \dfrac{y^2}{y+1}$ as local or global minimums or maximums.

15. Find the critical points of $k(x) = e^{-x}\cos(x)$ if $\dfrac{-\pi}{2} \le x \le \dfrac{\pi}{2}$ and the interval on which f is decreasing for the constrained values of x.

16. Find the critical points of $f(x) = x - \ln(x)$ and the intervals on which f is increasing.

17. Determine the intervals on which $y = 2x^2 + \ln(x)$ is concave up or down and find the points of inflection.

18. Classify all critical points of $g(x) = \sin^2(x) + \cos(x)$ for $0 \leq x \leq \pi$ as minimums, maximums, or saddle points.

19. Classify all critical points of $h(x) = xe^{-x^2}$ as minimums, maximums, or saddle points. Then find all inflection points.

20. Classify all critical points of $f(x) = x^3 \ln(x)$ as minimums, maximums, or saddle points. Then find all inflection points.

21. Classify all critical points of $p(x) = 2x^4 - 3x^2 + 2$ as local or global minimums and maximums, or as saddle points. Then find the inflection points and the interval(s) where p is concave down.

22. Sketch the graph of a function f so that $f'(x) > 0$ for all x, $f''(x) > 0$ for $x < 0$, and $f''(x) < 0$ for $x > 0$.

23. CAS problem (3 points): use a CAS device to solve the following problems. Submit a printed copy of the device's solution and your corresponding commands.

    a) Graph $f(x) = \dfrac{x^{2/3}}{1 + x + x^4}$.

    b) Find all of the critical points for $f(x) = \dfrac{x^{2/3}}{1 + x + x^4}$. Determine which correspond to maximums and which to minimums. Approximate your answers using four digits. Be sure to inspect the graph to verify that you have all of the critical points!

Brief answers

1. $\dfrac{\pi}{150}$
2. $\dfrac{1}{135}$
3. -0.03
4. $L(\theta) = \dfrac{1}{2} - \dfrac{\pi}{4} + \theta$
5. $L(x) = ex - e$
6. $L(x) = 1 + x$.
7. $\dfrac{2}{3}\%$
8. 0.06
9. $c = 4$
10. $c = \ln(e - 1)$.
11. $x = -1$, and $x = 3$
12. Minimum is $y(\pi/3) = \pi/3 - \sqrt{3}$; maximum is $y(5\pi/3) = 5\pi/3 + \sqrt{3}$.

13. maximum $= g(\ln(3/2)) = 9/4$; minimum is $g(10) = 3e^{10} - e^{20}$.

14. The critical points are -2, a local maximum, and 0, a local minimum.

15. $x = -\frac{\pi}{4}$ is the critical point and k is decreasing on $(-\pi/4, \pi/2]$.

16. $x = 1$ is the only critical point, f is increasing on $(1, \infty)$.

17. Concave down on $(0, 0.5)$, concave up on $(0.5, \infty)$, inflection point at $x = 0.5$.

18. $x = 0$ and $x = \pi$ are minimums, $x = \pi/3$ is a maximum.

19. $\frac{\sqrt{2}}{2}$ is a maximum, $-\frac{\sqrt{2}}{2}$ is a minimum. 0, $\pm\sqrt{\frac{3}{2}}$ are inflection points.

20. $x = e^{-1/3}$ is a minimum, $x = e^{-5/6}$ is an inflection point.

21. $\pm\frac{\sqrt{3}}{2}$ are global minimums, 0 is a local maximum, ± 0.5 are inflection points, concave down on the interval $(-0.5, 0.5)$.

22. Any graph similar that of $\arctan(x)$.