

1. Compute  $\frac{d}{dx} \left( \tan^{-1}(\ln(x+1)) + e^{\sin^2(x)} \right) \Big|_{x=0}$
2. Evaluate  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x^3 - 7x - 6}$
3. Evaluate  $\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{\sin(5x)}$
4. Evaluate  $\lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$
5. Evaluate  $\lim_{x \rightarrow -\infty} \frac{7x^2 + 4x}{9 - 3x^2}$
6. Evaluate  $\lim_{x \rightarrow 0} \frac{\sin(2x)}{\sin(7x)}$
7. Evaluate  $\lim_{x \rightarrow \pi/2} \sec(x) - \tan(x)$
8. Evaluate  $\lim_{x \rightarrow 0} \frac{e^{2x} - 1 - x}{x^2}$
9. Evaluate  $\lim_{x \rightarrow 0} \frac{\arcsin(x)}{x}$
10. Evaluate  $\lim_{x \rightarrow \infty} \left( 1 + \frac{1}{x} \right)^x$ .
11. Sketch the graph of  $y = x^3 - 3x + 5$  after making sign charts for the first two derivatives. Use a graphing tool such as Wolfram Alpha, a CAS, or a graphing calculator to check your answer.
12. Sketch the graph of the everywhere continuous function  $f(x)$  for which  $f'(x) < 0$  for  $6 < x < 8$  and  $x > 8$ ,  $f'(8)$  does not exist, and  $f'(x) > 0$  for  $x < 6$ ; while  $f''(x) > 0$  for  $8 < x < 12$  and negative outside the interval  $[8, 12]$ .
13. Sketch the graph of the continuous even function  $k(x)$  for which  $\lim_{x \rightarrow \infty} k(x) = 0$ ;  $k'(x) < 0$  for  $x > 0$  and  $k''(x) < 0$  for  $0 < x < \frac{1}{\sqrt{3}}$ , but  $k''(x) > 0$  for  $x > \frac{1}{\sqrt{3}}$ .
14. Sketch the graph of the function  $h(x)$  for which  $\lim_{x \rightarrow 0^-} h(x) = -\infty$ ;  $\lim_{x \rightarrow 0^+} h(x) = \infty$ ;  $\lim_{x \rightarrow 1^-} h(x) = -\infty$ ;  $\lim_{x \rightarrow 1^+} h(x) = \infty$ ;  $\lim_{x \rightarrow -\infty} h(x) = 0$ ;  $\lim_{x \rightarrow \infty} h(x) = 0$ ;  $h'(x) < 0$  for all  $x$ ;  $h''(x) > 0$  for  $0 < x < 0.5$  and  $x > 1$  but negative otherwise.
15. Sketch the graph of the continuous function  $g(x)$  with domain  $x > 0$  for which  $\lim_{x \rightarrow 0^+} g(x) = \infty$ ;  $g'(x) < 0$  for  $0 < x < 1$  and  $g'(x) > 0$  for  $x > 1$ ; while  $g''(x) > 0$  for  $x > 0$ .
16. Sketch the graph of the continuous odd function  $f(x)$  for which  $f'(x) > 0$  for  $0 < x < 1$  and  $f'(x) < 0$  for  $x > 1$ ; while  $f''(x) < 0$  for  $0 < x < \sqrt{3}$  and  $f''(x) > 0$  for  $x > \sqrt{3}$ .
17. A cylindrical can with a bottom but not a top has volume equal to  $8 \text{ m}^3$ . What are the dimensions of the can that minimize surface area?
18. A poster of area  $5000 \text{ cm}^2$  has blank margins of width 5 cm on the top and bottom and 2 cm on the sides. Find the dimensions of the poster that maximize the printed area.

19. Find the equation of the line through the point  $(4, 12)$  so that the area of the triangle bounded by this line and the axes in the first quadrant has minimal area.
20. Let  $x$  and  $y$  be two positive numbers and  $k$  a constant so that  $x + y = k > 0$  and suppose  $n > 1$ . Show that by setting  $x = y$  that a)  $x^n + y^n$  is minimized, and b)  $x^n y^n$  is maximized.
21. All units in a 1000-unit apartment complex are occupied when the rent is \$1200 per month. The landlord knows from experience that a \$50 increase in rent per month will result in a loss of ten rented units. However, each occupied unit results in a \$200 maintenance charge for the landlord. What rent will maximize the landlord's profit? How many units will be rented at this price?
22. Approximate  $\sqrt{11}$  to three decimal places using Newton's method and any calculating device. Hint: Estimate a zero for  $f(x) = x^2 - 11$ .
23. CAS problem (3 points): use a CAS device to solve the following problem. Submit a printed copy of the device's solution and your corresponding commands.  
Use Newton's method to estimate the solutions for  $x^3 - 9x - 4 = 0$  using three different initial guesses:  $g_0 = 1.5$ ,  $g_0 = 1.73$ , and  $g_0 = 2$ . Explain why the three estimates are different and decide if the estimates are close to a solution. Use a graph to help your explanation.

### Brief answers

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|---|---|
| 1. 1  | 13. Looks like $y = \frac{1}{1 + x^2}$  |
| 2. $1/80$   | 14. Looks like $y = x^{-1} + (x - 1)^{-1}$                                    |
| 3. 0  | 15. Looks like $y = x^2 - 2 \ln(x)$ .   |
| 4. 0  | 16. Looks like $y = \frac{x}{\sqrt{2}e^{x^2/2}}$ .                            |
| 5. $-7/3$   | 17. Radius = $\frac{2}{\sqrt[3]{\pi}}$ ; Height = $\frac{2}{\sqrt[3]{\pi}}$   |
| 6. $2/7$  | 18. $20\sqrt{5}$ by $50\sqrt{5}$  |
| 7. 0  | 19. $y = -3x + 24$  |
| 8. DNE  | 20. Hint: a) use second derivative test;<br>b) use the first derivative test. |
| 9. 1  | 21. \$3200 per month, 600 units   |
| 10. $e$   | 22. 3.317   |
| 11. Hint: $y' < 0$ in $(-1, 1)$ , $y'' > 0$ for $x > 0$ . |   |
| 12. Looks like $y = x\sqrt[3]{8 - x}$ .                   |   |