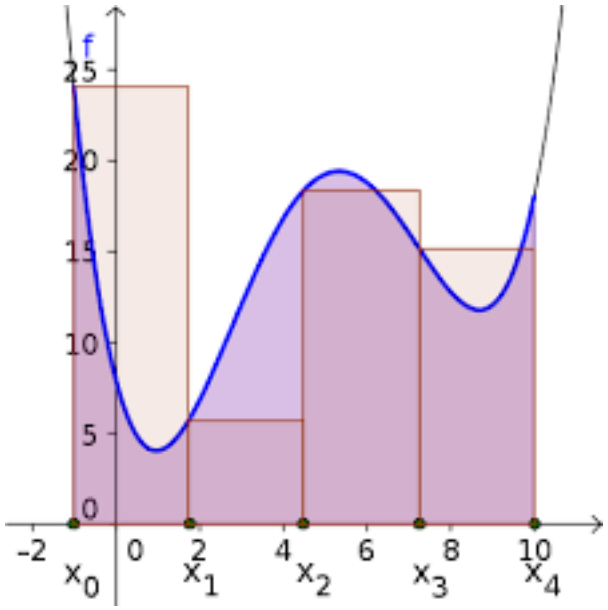
25 points | Math 150 | Homework 7 | Secions 5.1 - 5.4 | 23 problems

1. Write $I = \int_0^{\pi} \sin(x)dx$ as a limit of a Riemann sum using the right endpoint of each subinterval.

2. Find a definite integral equal to $\lim_{N\to\infty} \frac{3}{N} \sum_{j=1}^{N} \frac{1}{1+\frac{3j}{N}}$.

3. The partition below estimates $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\frac{b-a}{n}$. What are a, b, n, and x_3^*? Estimate the Riemann sum corresponding to this partition.



4. Compute $\int_0^5 \sqrt{25-x^2}\,dx$ using geometry. Sketch a graph to defend your answer.

5. Compute $\int_{-3}^{3} 2-|x|\,dx$ using geometry. Sketch a graph to defend your answer.

6. Draw a graph and use geometry to evaluate $\int_0^6 f(x)\,dx$ if $f(x) = \begin{cases} -\sqrt{1-(x-1)^2} & 0 \le x \le 2 \\ \sqrt{4-(x-4)^2} & 2 < x \le 6 \end{cases}$.

7. Draw a graph to determine the sign of $\int_{-2}^{1} x^3 dx$.

8. Evaluate $\int_{-2}^{2} x^5 \cos(x^2)\,dx$. Defend your answer.

9. Calculate $\int_2^4 f(x)\,dx$ if $\int_0^1 f(x)\,dx = 1$, $\int_0^2 f(x)\,dx = 6$, and $\int_1^4 f(x)\,dx = 9$.

10. Express $\displaystyle\int_2^9 f(x)\,dx - \int_2^5 f(x)\,dx$ as a single integral.

11. Calculate $\displaystyle\int_2^5 2f(x) - 3\,dx$ if $\displaystyle\int_1^b f(x)\,dx = 1 - b^{-1}$. Hint: use linearity and the splitting theorem.

12. Find positive constants a so that $\displaystyle\int_a^{a+1} x^5\,dx < \int_a^{a+1} x^4\,dx$ but $\displaystyle\int_{a+1}^{a+2} x^5\,dx > \int_{a+1}^{a+2} x^4\,dx$. Defend your answer. Hint: use the comparison theorem.

13. Find the general antiderivative of $2x^{-4} + 12x^{-1} + 2\cos(2x) - 9\sin(x)$.

14. Evaluate $\displaystyle\int \frac{1}{\sqrt[3]{t}}\,dt$.

15. Evaluate $\displaystyle\int 12\sec(\theta)\tan(\theta)\,d\theta$.

16. Evaluate $\displaystyle\int 8x - 4e^{5-2x}\,dx$.

17. Evaluate $\displaystyle\int_1^9 \frac{3y+1}{\sqrt{y}}\,dy$.

18. Evaluate $\displaystyle\int_0^{\pi/6} \sec^2\left(3x - \frac{\pi}{6}\right)\,dx$.

19. Evaluate $\displaystyle\int_0^3 e^{1-6t}\,dt$.

20. Evaluate $\displaystyle\int_0^{\pi} |\cos(x)|\,dx$.

21. Evaluate $\displaystyle\int_c^{5c} \frac{dx}{x}$ if $c > 0$ is a constant.

22. Calculate $\displaystyle\int_{-2}^4 f(x)\,dx$ if $f(x) = \begin{cases} 3x^2 - 2x & x \le 2 \\ x^3 & x > 2 \end{cases}$.

23. CAS problem (3 points): use a CAS device to solve the following problem. Submit a pdf copy of the device's solution and your corresponding commands.

    Use a Riemann sum to give an estimate, using four digits, of $\displaystyle\int_0^{\pi} \cos(x^2)\,dx$. Use 100 subintervals of equal length and the right endpoint for x_i^* .

Brief answers

1. $\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{\pi}{n} i\right) \frac{\pi}{n}$
2. $\int_1^4 \frac{1}{x}\, dx$
3. $a = -1$, $b = 10$, $n = 4$, $x_3^* \approx 4.5$ or 6.5, Sum ≈ 176
4. $\frac{25\pi}{4}$
5. 3
6. $\frac{3\pi}{2}$
7. negative.
8. 0
9. 4
10. $\int_5^9 f(x)\, dx$
11. $\frac{-42}{5}$
12. $a = 0$
13. $\frac{-2x^{-3}}{3} + 12\ln|x| + \sin(2x) + 9\cos(x) + C$
14. $1.5t^{2/3} + C$
15. $12\sec(\theta) + C$
16. $4x^2 + 2e^{5-2x} + C$
17. 56
18. $\frac{4\sqrt{3}}{9}$
19. $\frac{e - e^{-17}}{6}$
20. 2
21. $\ln(5)$
22. 76