25 points | Math 150 | Homework 9 | Secions 6.1, 7.1 | 23 problems

1. Evaluate $\displaystyle\int_0^1 \frac{(\tan^{-1}(x))^3}{1+x^2}\,dx$

2. Evaluate $\displaystyle\int \frac{\arcsin(t)}{\sqrt{1-t^2}}\,dt.$

3. Sketch the region enclosed by the graphs of $f(x) = 60+3x-3x^2$ and $g(x) = 3x^2-15x$, and compute its area.

4. Find the area to the right of $x = y^2 - 5$ and to the left of $x = 3 - y^2$.

5. Sketch the region enclosed by $x + y = 4$, $y = x$, and $y + 3x = 4$; and then compute its area.

6. Sketch the region enclosed by $y = \cos(x)$, $y = \cos(2x)$, $x = 0$, and $x = \dfrac{2\pi}{3}$, and then compute its area.

7. Sketch the region enclosed by $y = e^x$, $y = e^{-x}$, and $y = 2$, and then compute its area.

Solve the following using integration by parts.

8. Evaluate $\displaystyle\int xe^{5x+2}\,dx.$

9. Evaluate $\displaystyle\int x^3e^x\,dx.$

10. Compute $\displaystyle\int_1^e x^3\ln(x)\,dx$

11. Compute $\displaystyle\int_0^\pi x^2\sin(x)\,dx.$

12. Evaluate $\displaystyle\int e^{-x}\sin(x)\,dx.$

13. Evaluate $\displaystyle\int p\ln(p)\,dp.$

14. Compute $\displaystyle\int_e^{e^4} (\ln(x))^2\,dx.$

15. Evaluate $\displaystyle\int x^2\cosh(x)\,dx.$

Solve the following using substitution, integration by parts, or both.

16. Compute $\displaystyle\int \frac{\tan(\sqrt{x})}{\sqrt{x}}\,dx.$

17. Evaluate $\displaystyle\int \frac{\ln(\ln(x))\ln(x)}{x}\,dx.$

18. Compute $\displaystyle\int_0^3 xe^{4x}\,dx.$

19. Compute $\displaystyle\int_0^1 x3^x\,dx.$

20. Evaluate $\displaystyle\int \frac{x\,dx}{\sqrt{x+1}}$ using substitution.

21. Evaluate $\displaystyle\int \frac{x\,dx}{\sqrt{x+1}}$ using parts.

22. The answers from the last two problems should be equal. Prove it. That is, show that

$$\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} = -\frac{4}{3}(x+1)^{3/2} + 2x(x+1)^{1/2}$$

23. CAS problem (3 points): use a CAS device to solve the following problem. Submit a pdf copy of the device's solution and your corresponding commands.

    Find the area between the curves $y = \dfrac{\sin(2x)}{\sqrt{5-\sin(x)}}$ and $y = -x$ for $0 \le x \le 1$. Give a four digit approximation. Draw a graph of these curves with the area between them shaded.

Brief answers

1. $\dfrac{\pi^4}{4^5}$
2. $\dfrac{(\sin^{-1}(t))^2}{2} + C$
3. 343
4. $\dfrac{64}{3}$
5. 2
6. $\dfrac{3\sqrt{3}}{4}$
7. $\ln(16) - 2$
8. $\dfrac{xe^{5x+2}}{5} - \dfrac{e^{5x+2}}{25} + C$
9. $e^x(x^3 - 3x^2 + 6x - 6) + C$
10. $\dfrac{3e^4 + 1}{16}$
11. $\pi^2 - 4$
12. $\dfrac{-e^{-x}(\cos(x) + \sin(x))}{2} + C$
13. $\dfrac{p^2 \ln(p)}{2} - \dfrac{p^2}{4} + C$
14. $10e^4 - e$
15. $x^2 \sinh(x) - 2x \cosh(x) + 2 \sinh(x) + C$
16. $2 \ln|\sec(\sqrt{x})| + C$
17. $\dfrac{(\ln(x))^2 \ln(\ln(x))}{2} - \dfrac{(\ln(x))^2}{4} + C$
18. $\dfrac{11e^{12} + 1}{16}$
19. $\dfrac{3\ln(3) - 2}{(\ln(3))^2}$
20. $\dfrac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + C$
21. $-\dfrac{4}{3}(x+1)^{3/2} + 2x(x+1)^{1/2} + C$
22. Hint: factor $(x+1)^{1/2}$ from both solutions.