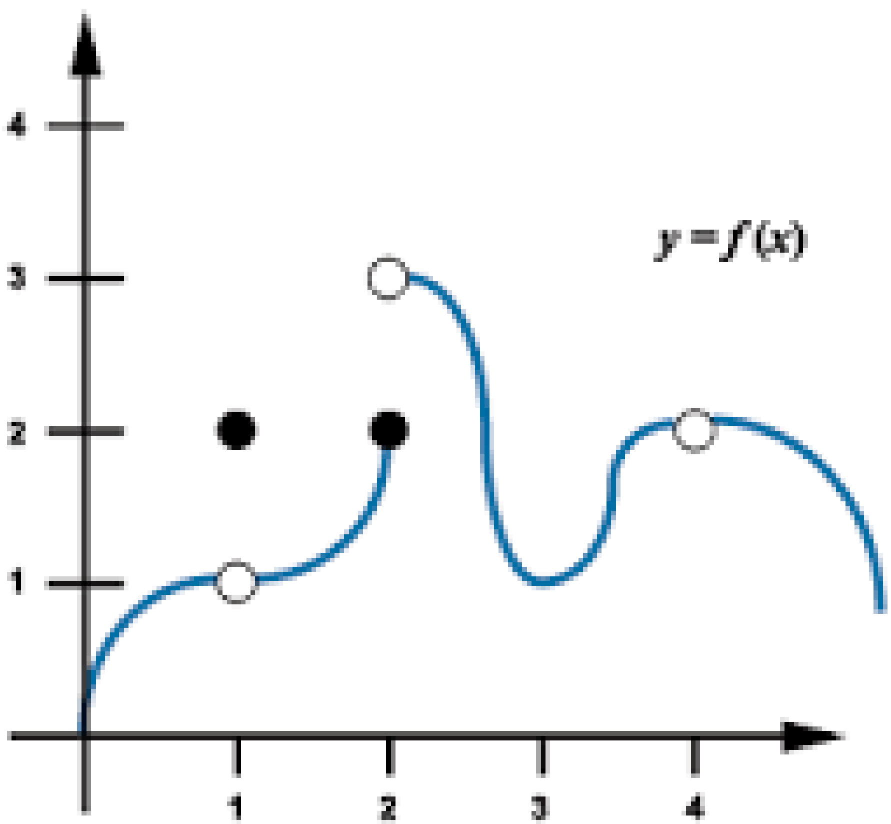
Introduction

Our Calculus class describes and calculates **limits**, **derivatives**, and **integrals**.

Limits

<u>Intuitive Definition</u>: $\lim_{x \to a} f(x) = L$ means $f(x)$ gets close to L as x gets close to, but **not** equal to, a.

Alternate notation: $f(x) \to L$ as $x \to a$.



$\lim_{x \to 1} f(x) =$ ________________ $\lim_{x \to 2} f(x) =$ ________________

$\lim_{x \to 3} f(x) =$ ________________ $\lim_{x \to 4} f(x) =$ ________________

Derivatives

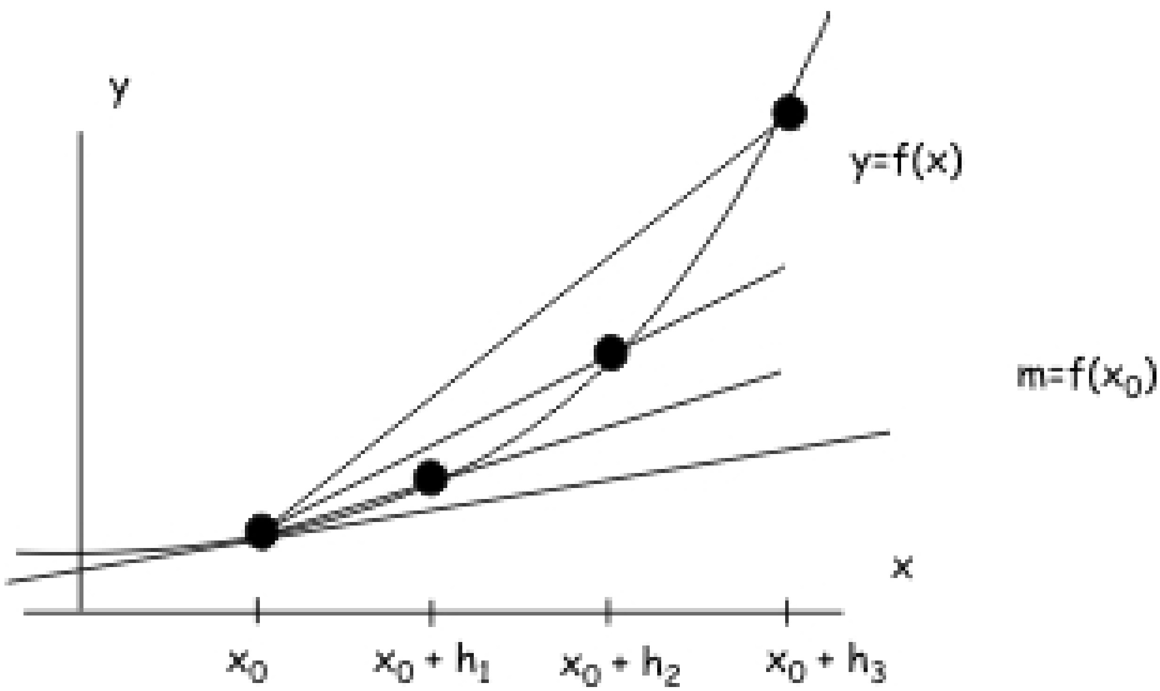
Rigorous Definition: The derivative of $f(x)$ at $x = a$ is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Alternate notation: $\left.\frac{df}{dx}\right|_{x=a}$.

Intuitive Definition:
1) The slope of the tangent line at $x = a$.
2) The instataneous rate of change of $f(x)$ at $x = a$.

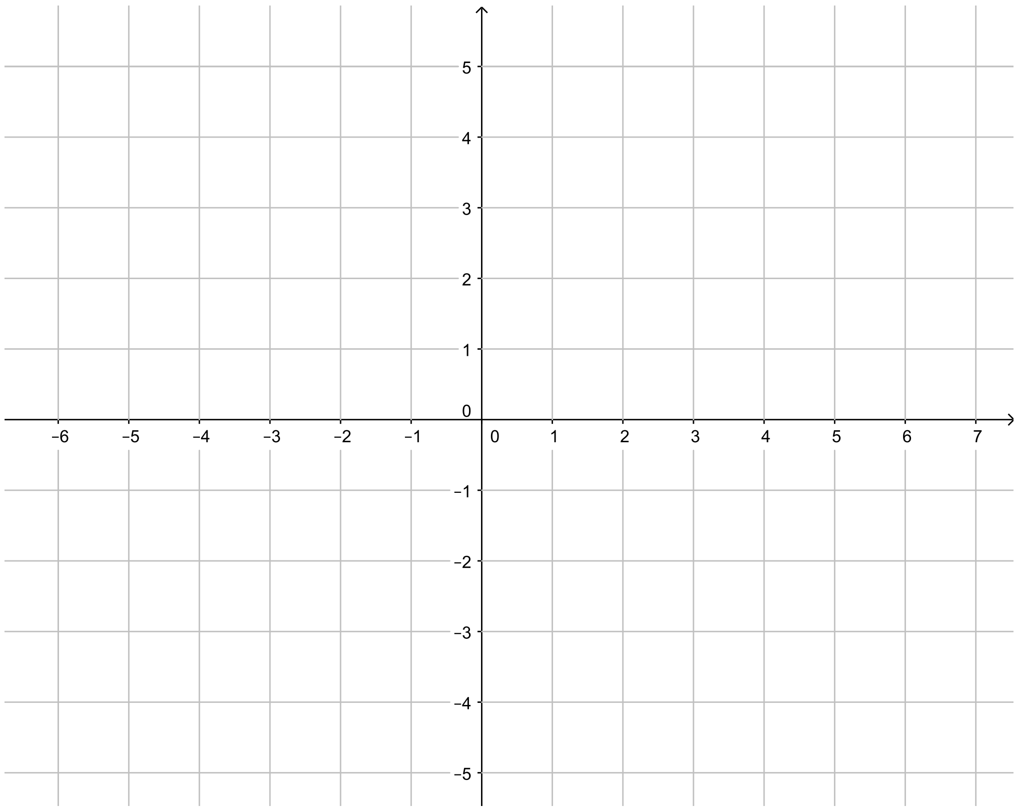
$\frac{df}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ is the derivative function; each output is the drivative of f for the corresponding input. We often call the derivative function "the derivative of f". '



The average rate-of-change of $f(x)$ over the interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$. This is a slope of a secant line.

What is the average rate of change of $g(x) = \ln(x)$ over the interval $[e, e^3]$?

Example: Jim is jumping on a trampoline, and $s(t) =$ the number of meters from equilibrium (the level of the tampoline when no one is on it) at elapsed seconds t. A care-free study shows $s(t) = -3\sin(t)$. Sketch the graph of $s(t)$.



Find the average velocity over the time interval $[0, 3\pi/2]$. Include units.

The average **speed** is the absolute value of the average velocity. What is the average speed over the time interval $[0, \pi/2]$? Include units.

Use the graph to estimate the instantaneous velocity at $t = 0$. You have just estimated $s'(0)$. What are the units?

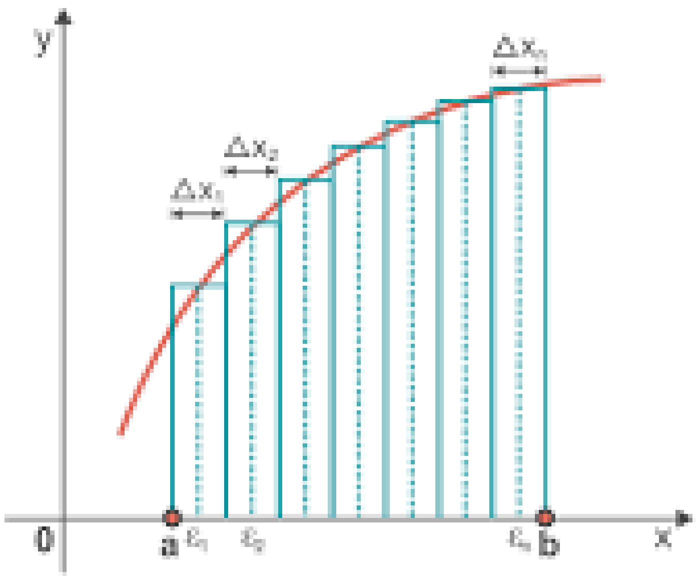
Integrals

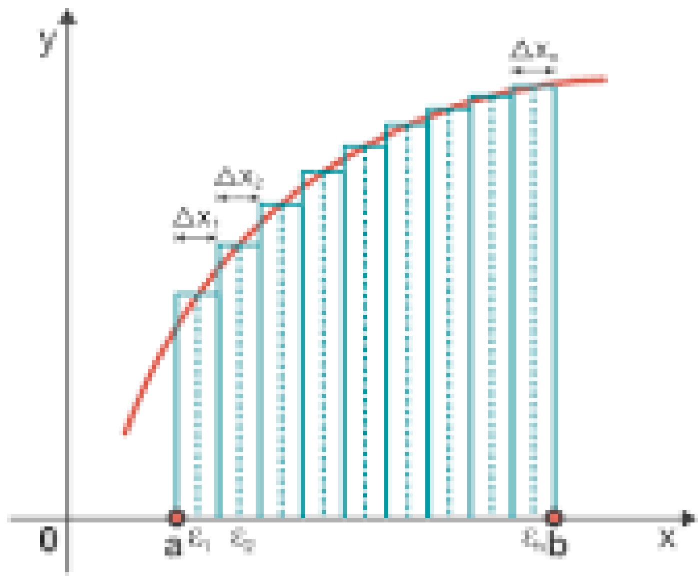
Rigorous Definition: The integral of $f(x)$ over the interval $[a, b]$ is

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

where $x_0 = a$, $x_n = b$, and x_i^* is a point in the ith subinterval. We also require $\Delta x \to 0$ as $n \to \infty$ where $\Delta x = max(\Delta x_i)$.

Intuitive Definition:
1) The **net** area under the curve.
2) The integral of $f'(x)$ over $[a, b]$ is the total change of $f(x)$ over $[a, b]$.





What are the units of $\int_0^{3\pi/2} s'(t)\,dt$ if $s(t)$ is the function from our trampoline example?