Using Algebra For Indeterminate Limits

Let $f(x) = \dfrac{x^2 - 81}{x - 9}$. Then $f(9) = \dfrac{0}{0}$ is an indeterminate form. Some other indeterminate forms are $\dfrac{\infty}{\infty}$, $\infty \cdot 0$, and $\infty - \infty$.

Consequently we cannot **yet** substitute to find $\lim\limits_{x \to 9} \dfrac{x^2 - 81}{x - 9}$. Sometimes (and often in this class) we can calculate the limit after we have used algebra to change the form of the function.

Example: Calculate $\lim\limits_{x \to 9} \dfrac{x^2 - 81}{x - 9}$.

Example: Calculate $\lim\limits_{h \to 0} \dfrac{(1 + h)^3 - 1}{h}$.

Example: Calculate $\lim_{x \to 8} \dfrac{\sqrt{x-4}-2}{x-8}$.

Example: Calculate $\lim_{x \to 0^+} \dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{x^2+x}}$.

Example: Calculate $\lim_{\theta \to \frac{\pi}{4}} \dfrac{\sin(\theta) - \cos(\theta)}{\tan(\theta) - 1}$.

Example: Calculate $\lim_{h \to a} \dfrac{\frac{1}{h} - \frac{1}{a}}{h - a}$.

Example: Calculate $\lim_{\theta \to 0} \dfrac{\sin(\theta)}{\theta}$.

No algebra we can do. We need the "Squeeze Theorem."