

Squeeze Theorem

If

1) $f(x) \leq g(x) \leq h(x)$ for $x \in (a, b)$

2) for $c \in (a, b)$, $\lim_{x \rightarrow c} f(x)$ and $\lim_{x \rightarrow c} h(x)$ both exist and equal L

then what can we conclude? Draw a picture and write the conclusion.

Example Calculate $\lim_{x \rightarrow 0} x \sin \left(\ln \left(\frac{1}{|x|} \right) \right)$

Prove

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

Hint: the area of a sector of a circle with radius r and angle θ is $\frac{\theta r^2}{2}$. This helps prove $\sin(x) < x < \tan(x)$ if x is small and positive.

Calculate

$$\lim_{x \rightarrow 0} \frac{\sin(5x) \sin(2x)}{\sin(3x) \sin(5x)}$$