Limits at Infinity

Use intuition to calculate the following limits.

$$\lim_{x \to \infty} \frac{4}{x+5}$$

$$\lim_{x \to -\infty} \frac{4}{x+5}$$

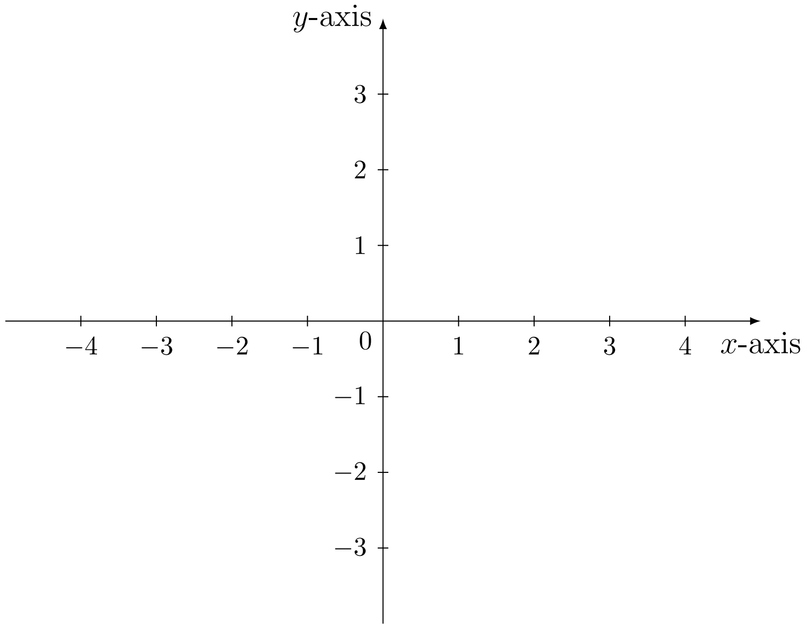
Intuitively

$\lim_{x \to \infty} f(x) = L$ iff $f(x)$ gets close to L as x becomes larger and larger positively.

Similarly,

$\lim_{x \to -\infty} f(x) = L$ iff $f(x)$ gets close to L as $x < 0$ has $|x|$ that gets larger and larger.

Graph $f(x) = \dfrac{4}{x+3}$. What are the horizontal and vertical asymptotes? Write limits to express the function's behavior near these asymptotes.



Calculate the limit of $f(x)$ as $x \to \pm\infty$ and then find all vertical and horizontal asymptotes. Calculations are easier to see if we divide top and bottom by the largest power of the unknown.

1) $f(x) = \dfrac{3x^2 + 20x + 9}{4x^2 + 9}$

2) $f(x) = \dfrac{8x^3 - x^2}{4x^4 - 9}$

Find the limits. Defend your answer with work.

$$\lim_{t \to -\infty} \frac{4 + 6e^{2t}}{5 - 9e^{3t}} \qquad \text{vs.} \qquad \lim_{t \to \infty} \frac{4 + 6e^{2t}}{5 - 9e^{3t}}$$

$$\lim_{x \to \infty} \left(\frac{1}{x} - \frac{1}{x+2} \right)$$

Find the limits. Defend your answer with work.

$$\lim_{x\to\infty} \tan^{-1}(x) \qquad \text{and} \qquad \lim_{x\to-\infty} \tan^{-1}(x)$$

$$\lim_{y\to\infty} \left(\ln\left(\sqrt{5y^2+2}\right) - \ln(y)\right) \qquad \text{vs.} \qquad \lim_{y\to-\infty} \frac{\sqrt{5y^2+2}}{y}$$