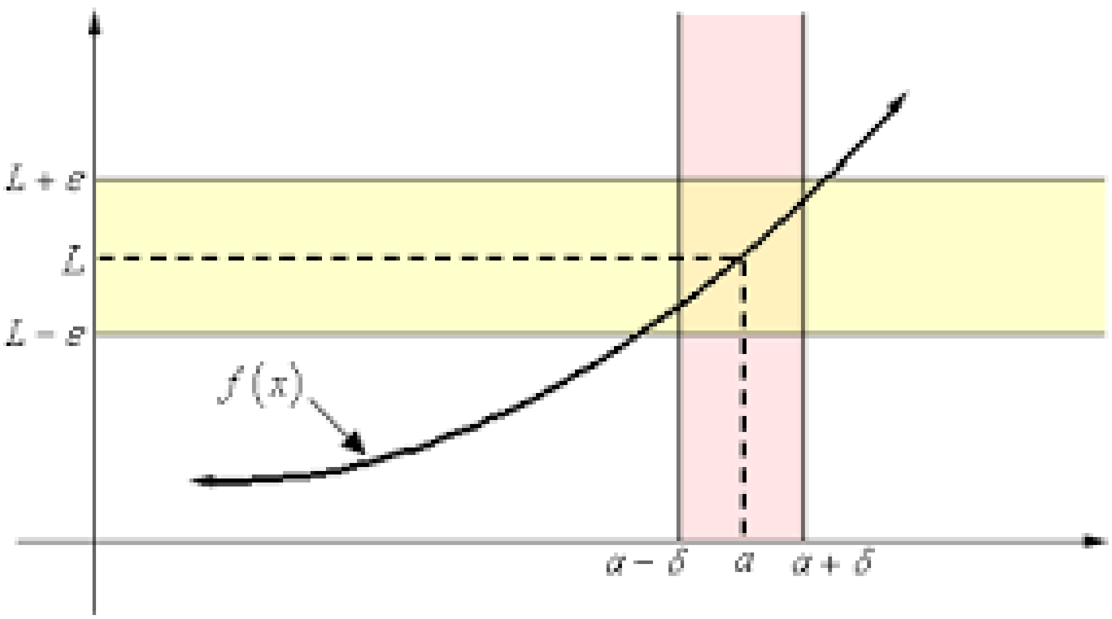
ϵ - δ Definition of Limit

Rigorous Definition: $\lim_{x \to a} f(x) = L$ means that for every $\epsilon > 0$ there exists $\delta > 0$ so that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.



Let $f(x) = 4x + 3$. For an arbitrary $\epsilon > 0$, find a corresponding δ in terms of ϵ so that $|f(x) - 11| < \epsilon$ whenever $|x - 2| < \delta$. Once finding that such a δ exists, you have proven what limit exists?

Find all possible $\delta > 0$ so that $x \in [2 - \delta, 2 + \delta]$ implies $f(x) \in [10.99, 11.01]$.

Let $g(x) = 2x - 1$. For an arbitrary $\epsilon > 0$, find a corresponding δ in terms of ϵ so that $|g(x) + 11| < \epsilon$ whenever $|x + 5| < \delta$. Once finding that such a δ exists, you have proven what limit exists?

Find all possible $\delta > 0$ so that $x \in [-5 - \delta, -5 + \delta]$ implies $g(x) \in [-11.5, -10.5]$.

Tactics for finding a delta when the function is nonlinear to prove $\lim_{x \to a} f(x) = L$.

1) Write $|f(x) - L|$ in terms of $|x - a|$ so that $|f(x) - L| = |p(x)||x - a|$.

2) Find a constant k so that there is an upper bound constant M for $|p(x)|$ if $0 < |x - a| < k$. Then $|f(x) - L| \leq M|x - a|$ when $0 < |x - a| < k$.

3) Set $\delta = \min\left(k, \frac{\epsilon}{M}\right)$. This implies $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$ and so we are done.

Example: Let $h(x) = 3x^2 + x$. For an arbitrary $\epsilon > 0$, find a corresponding δ in terms of ϵ so that $|h(x) - 4| < \epsilon$ whenever $|x - 1| < \delta$. Once finding that such a δ exists, you have proven what limit exists?

Find one possible $\delta > 0$ so that $x \in [1 - \delta, 1 + \delta]$ implies $h(x) \in [3.99, 4.01]$.