

Product and Quotient Rule (3.3)

Product Rule

$$(fg)' = f'g + fg'$$

Example Find the derivative of $h(x) = 2e^x(4x^5 - \sqrt{x})$.

Example Find the derivative of $p(t) = (4e^t - t^2)(t^{1/3} - 1)$.

Proof:

$$\begin{aligned}(fg)(x+h) - (fg)(x) &= (fg)(x+h) - f(x+h)g(x) + f(x+h)g(x) - (fg)(x) \\ &= f(x+h)(g(x+h) - g(x)) + (f(x+h) - f(x))g(x)\end{aligned}$$

so,

$$\frac{(fg)(x+h) - (fg)(x)}{h} = f(x+h)\frac{(g(x+h) - g(x))}{h} + \frac{(f(x+h) - f(x))}{h}g(x) \rightarrow f(x)g'(x) + f'(x)g(x)$$

as $h \rightarrow 0$ using the fact that limits distribute over addition and multiplication and f differentiable implies it is also continuous.

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

Example Find $\frac{dy}{dx}$ if $y(x) = \frac{x^4 + e^x}{2x + 1}$.

Example Simplify $\frac{d}{dt} \left(\frac{xt-4}{t^2-x} \right)$, where x is constant.

Example Find $q'(0)$ if $q(x) = \frac{f(x)g(x)}{h(x)k(x)}$ and

$f(0)$	$f'(0)$	$g(0)$	$g'(0)$	$h(0)$	$h'(0)$	$k(0)$	$k'(0)$
1	2	3	4	-4	-3	-2	-1

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Example Find $f'(0)$ if $f(x) = (1 + x + x^{5/3})\frac{3x^5 + 5x^4 + 5x + 1}{8x^9 - 7x^4 + 1}$. Hint: Substitute 0 for x after differentiating, and give each factor a function name.