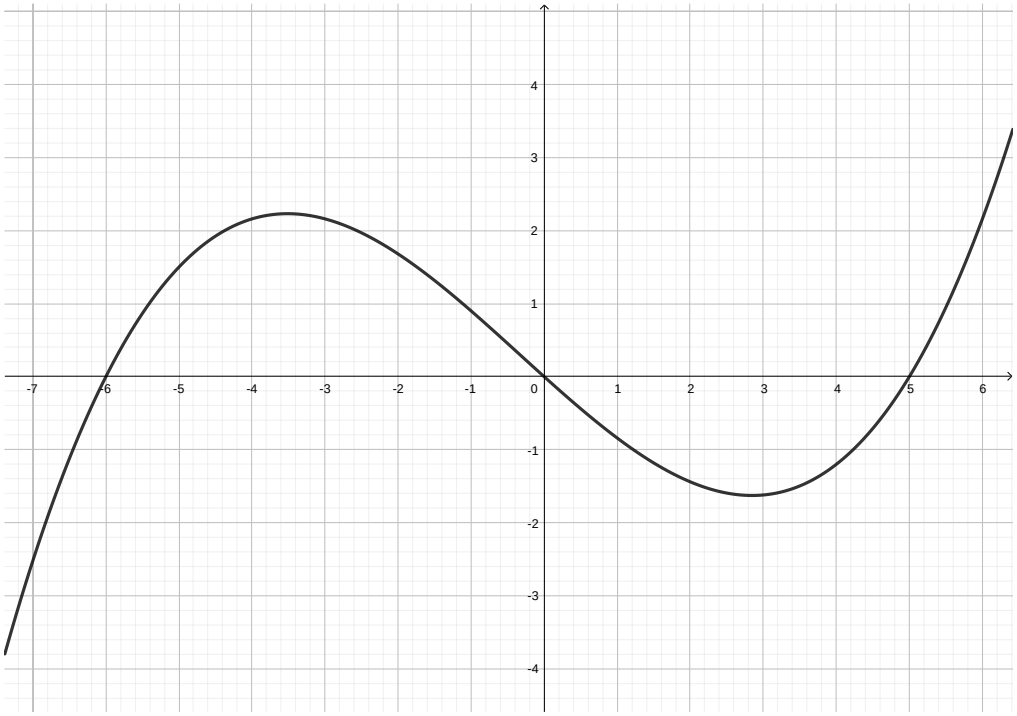
Higher Order Derivatives (3.5)

The graph of $y = f(x)$ is shown below. Where is $f'(x) > 0$? Where is $f'(x) < 0$?



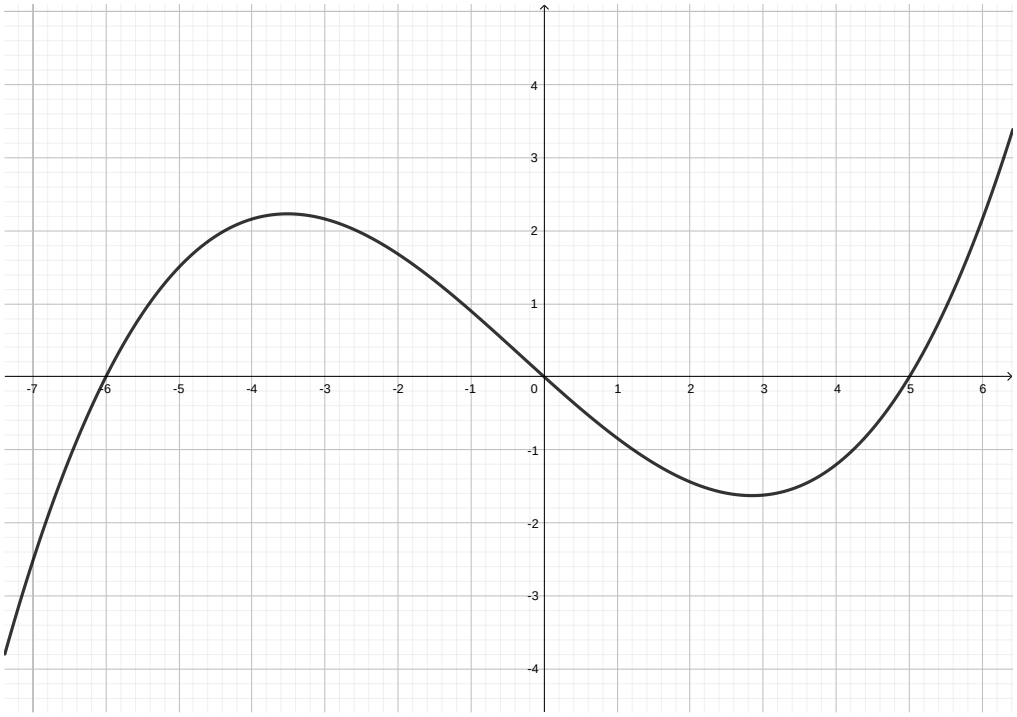
The outputs of $f(x)$ are **increasing** on an interval if $f'(x) > 0$ and **decreasing** if $f'(x) < 0$.

What does the derivative of the derivative tell us about the graph? This refers to "concavity".

Definition The **second derivative of** $f(x)$ is $\dfrac{d\,f'(x)}{dx}$.

Notation: $f''(x)$ or $\dfrac{d^2f}{dx^2}$. When evaluating at $x = a$ we write $f''(a)$ or $\left.\dfrac{d^2f}{dx^2}\right|_{x=a}$ or just $\left.\dfrac{d^2f}{dx^2}\right|_{a}$.

The graph of $y = f(x)$ is **concave up** over an interval if $f''(x) > 0$ on that interval, and **concave down** over the interval if $f''(x) < 0$. Determine where $f''(x)$ is positive and negative for the graph of $y = f(x)$ below. Notice over which intervals $f'(x)$ is increasing or decreasing.



Example Find the second derivative of $h(x) = xe^x$. Where is $y = h(x)$ concave up?

Example Find the second derivative of $g(t) = 4t^3 - 9\sqrt{t} + t^{-9/5}$.

Definition: The **nth order derivative of** $f(x)$ is denoted $f^{(n)}(x)$ or $\dfrac{d^n f(x)}{dx^n}$ and is defined recursively by $f^{(n+1)}(x) = \left(f^{(n)}(x)\right)'$.

Example Find the third derivative of $f(x) = x^{1/3}$. How do the domains of $f(x)$ and $f^{(3)}(x)$ compare?

Example Calculate $\left.\dfrac{d^4\left(5t^9 - t^5\right)}{dt^4}\right|_{t=1}$.

Example Find a general formula for $f^{(n)}(x)$ if $f(x) = x^2 e^x$.