Rates of Change (3.4)

$s(t)$ often denotes the position of a particle at time t. In this case

1) $s'(t)$ is the **instantaneous velocity** (positve or negative.)

2) $|s'(t)|$ is the **speed** (nonnegative.)

3) $s''(t)$ is the **acceleration**.

Example The position of a rock thrown from the top of a cliff is $s(t) = -5t^2 + 40t + 100$ meters above sea level after t seconds have elapsed.

a) What is the average velocity of the rock from 0 to 5 seconds of elapsed time?

b) What is the velocity at 5 seconds? What about at $t = 1$ seconds?

c) Is the rock speeding up or slowing down at $t = 5$? How about at $t = 1$? How do you know?

Example Suppose the cost of making q hundred thousand loaves of bread is $C(q) = 5q^2 + 20q + 110$. $C'(q)$ is the **marginal cost** because, for large scale productions such as this one,

$$C'(q) \approx \frac{C(q + 10^{-5}) - C(q)}{10^{-5}}$$

the cost of making the next loaf of bread. Notice that 10^{-5} hundred thousand loaves of bread equals one loaf of bread.

What does $C'(5)$ represent in this context?

Example $q(t)$ = charge (Coulombs)on a capacitor at time t (seconds.)

t	1	2	3	4
q	10	8	4	0

a) Estimate $q'(2)$.

b) What does $q'(2)$ mean in this context?

As time allows:

Example $T(x)$ = temperature (°C) of a steel bar at position x centimeters from the left end of the bar. In this context, $T'(2) = 5$ means what?

This is the one-dimensional **temperature gradient**.