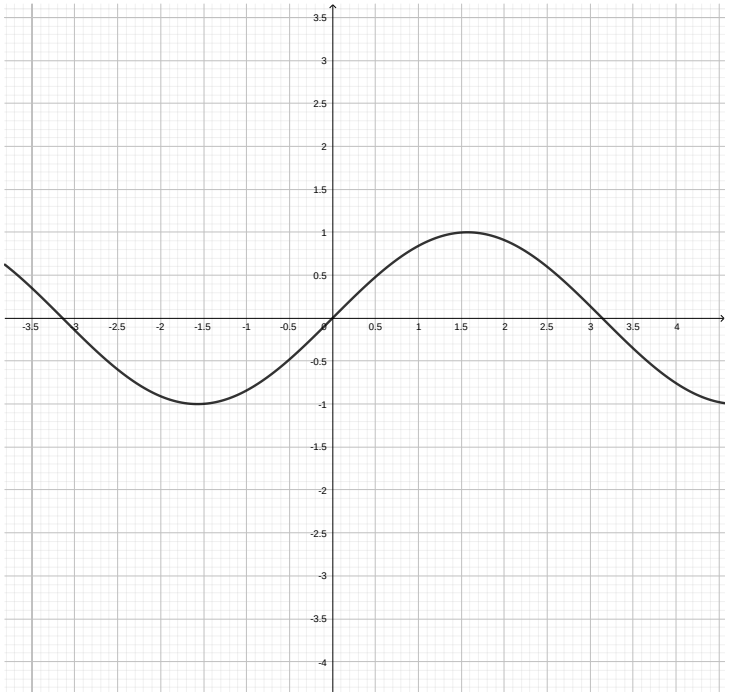
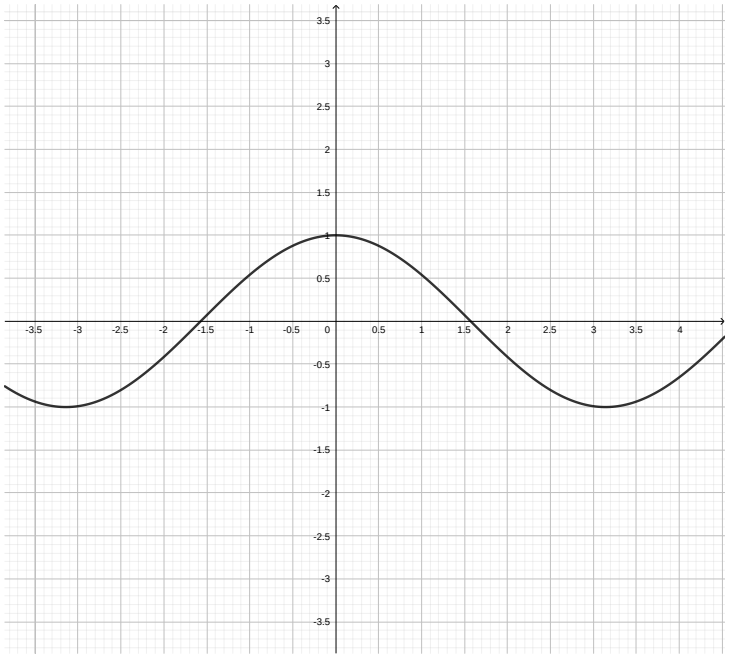
Derivatives of Trigonometric Functions (3.6)

Below is the graph of $y = \sin(x)$. Draw the graph of its derivative. What famous function do you think equals $\dfrac{d\sin(x)}{dx}$?



Below is the graph of $y = \cos(x)$. Draw the graph of its derivative. What famous function do you think equals $\dfrac{d\cos(x)}{dx}$?



From the graphs we correctly guess that

$$\frac{d\sin(x)}{dx} = \cos(x)$$

and that

$$\frac{d\cos(x)}{dx} = -\sin(x)$$

. To remember which has a minus sign you can remember the graphs above or "**sine is silent, cosine complains.**"

Let's prove these formulas algebraically. To do this your must remember the following.

1) $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$

2) $\lim_{x\to 0} \frac{\cos(x)-1}{x} = 0$

3) $\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

4) $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

Let's use the definition of the derivative to prove $\frac{d\sin(x)}{dx} = \cos(x)$.

Now let's use the definition of the derivative to prove $\frac{d\cos(x)}{dx} = -\sin(x)$.

We can use the quotient rule to find the derivative for the rest of the trigonometric functions.

Find $\frac{d\tan(x)}{dx}$.

Find $\dfrac{d\sec(x)}{dx}$.

Find $\dfrac{d\cot(x)}{dx}$.

Find $\dfrac{d\csc(x)}{dx}$.

To summarize we have:

$$\frac{d\sin(x)}{dx} = \cos(x)$$
$$\frac{d\cos(x)}{dx} = -\sin(x)$$
$$\frac{d\tan(x)}{dx} = \sec^2(x)$$
$$\frac{d\sec(x)}{dx} = \sec(x)\tan(x)$$
$$\frac{d\cot(x)}{dx} = -\csc^2(x)$$
$$\frac{d\csc(x)}{dx} = -\cot(x)\csc(x)$$

Notice once again that all the functions starting with the letter "c" include a minus sign in their derivatives.

Some practice problems follow.

If $f(x) = \dfrac{1}{\tan(x) + \sec(x)}$, find $f'(\dfrac{\pi}{4})$.

If $s(t) = 300 + 40\sin(t)$ is the position of a particle at time t, find the velocity, $v(t)$, and acceleration, $a(t)$, of the particle at any time t.