Logarithms and Hyperbolics (3.9)

Prove $\dfrac{d \ln(x)}{d\, x} = \dfrac{1}{x}$ using implicit differentiation.

Prove $\dfrac{d \ln|x|}{d\, x} = \dfrac{1}{x}$ using the chain rule.

Prove $y = x^r \implies y' = rx^{r-1}$.

Find the derivatives of the following functions.

1) $f(x) = \dfrac{\ln(x)}{x}$.

2) $g(x) = \ln(\tan(x))$.

3) $h(x) = \ln\left(\dfrac{x+1}{x^3+1}\right)$.

Find the derivatives of the following functions.

1) $f(x) = 5^x$.

2) $g(x) = x^x$.

3) $h(x) = \ln\left(x^{x^2}\right)$.

e^x is the most famous and useful function in mathematics, and so the **hyperbolic functions** are important too since they are made from exponentials.

Definitions

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

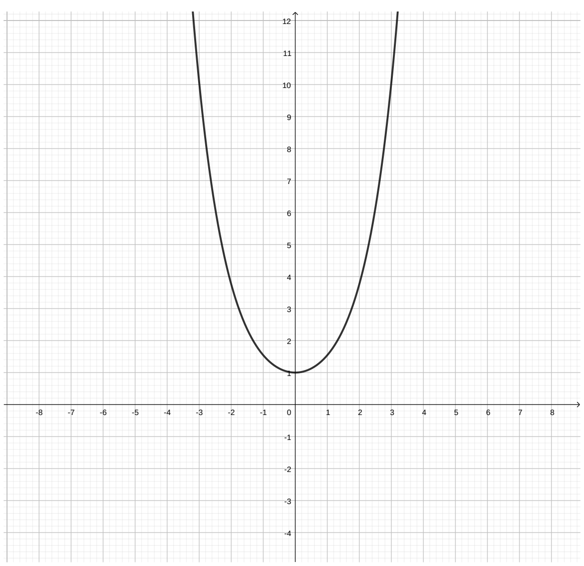
$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

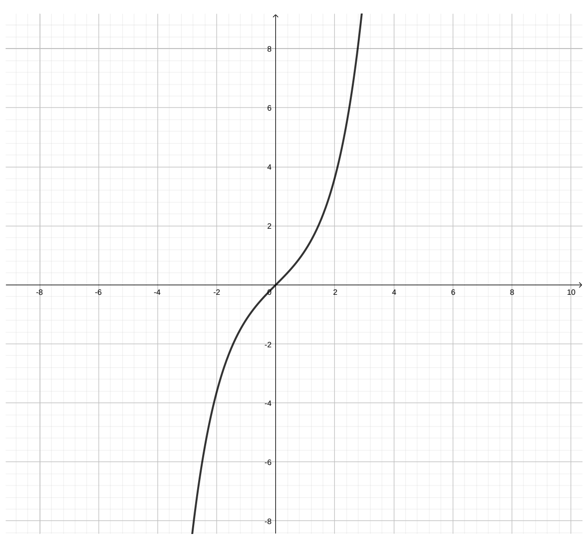
Notice that

$$\cosh(x) + \sinh(x) = e^x$$
$$\cosh(-x) = \cosh(x)$$
$$\sinh(-x) = -\sinh(x)$$

show that $\cosh(x)$ is the even part of e^x and $\sinh(x)$ is its odd part.

Which of the following graphs represent $\cosh(x)$ and $\sinh(x)$? Sketch their derivatives.





Why "hyperbolic?" Because $x^2 - y^2 = 1$ is a hyperbola, and $\cosh^2(x) - \sinh^2(x) = 1$. You should prove that. Compare this to trig functions defined on the circle $x^2 + y^2 = 1$; in that case $\cos^2(x) + \sin^2(x) = 1$.

Prove $\dfrac{d\,\sinh(x)}{d\,x} = \cosh(x)$.

Prove $\dfrac{d\,\cosh(x)}{d\,x} = \sinh(x)$.

Unlike trig functions, there are no minus signs in this derivative. For that reason, the hyperbolics are known to be a "sinch." International students, "sinch" is U.S. slang for "easy to do."

The other hyperbolic functions are defined the same way the trig functions are. For instance,

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

Define the other three hyperbolic functions.

Differentiate the following functions.

1) $y = 16^{\cos(2x)+\cosh(4x)}$.

2) $y = \sinh(\cosh^3(x))$.

3) $y = \ln(\coth(x))$.

As time allows.

4) $y = \big(\ln(\cosh(\sqrt{x}))\big)^5$.