Linear Approximations and Differentials (4.1)

Find the tangent line to the graph of $f(x) = e^x \cos(x)$ at $x = 0$.

In general, the tangent line to the graph of a function $y = f(x)$ at $x = 0$ is $y = f(0) + f'(0)x$. We say that $L(x) = f(0) + f'(0)x$ is the linear approximation for $f(x)$ at $x = 0$.

Find the tangent line to the graph of $f(x) = e^x \cos(x)$ at $x = a$.

In general, the tangent line to the graph of a function $y = f(x)$ at $x = a$ is $y = f(a) + f'(a)(x - a)$. We say that $L(x) = f(a) + f'(a)(x - a)$ is the **linear approximation** for $f(x)$ at $x = a$.

Find these famous linear approximations for the following famous functions at $x = 0$.

a) e^x

b) $\sin(x)$

c) $\cos(x)$

d) $\ln(1 + x)$

e) $(1 + x)^r$

Estimate $\ln(1.1)$.

If $f(x) = \dfrac{e^{-3x}}{\sqrt{1+x}}$, estimate $f(.1)$.

Use a linear approximation to estimate $f(x) = \dfrac{\sin(x)}{x}$ when x is close to zero.

Find the linear approximation for $g(x) = \dfrac{x+1}{x-1}$ at $x = 2$.

Recall the linear approximation for $f(x)$ at $x = a$ is $L(x) = f(a) + f'(a)(x - a)$.

We define the **differential** of $f(x)$ at $x = a$ to be $df = f'(a)(x - a)$.

If we let $x - a = \Delta x = dx$, then $df = f'(x)dx$.

Draw a picture labeling df and $\Delta f = f(x) - f(a)$ at $x = a$ on a graph of $f(x)$.

We see that

$$df \approx \Delta f$$

Use differentials to estimate the amount of paint needed for one coat of paint that is 0.05cm thick applied to a hemispherical dome with diameter 50 meters. Recall the volume of a sphere is $V = \frac{4}{3}\pi R^3$.

The radius of a circular disk is given as 24 ± 0.2 cm.

a) Use differentials to estimate the maximum error in calculating the area of the disk.

b) What is the **relative error**? What is the **percentage error**?