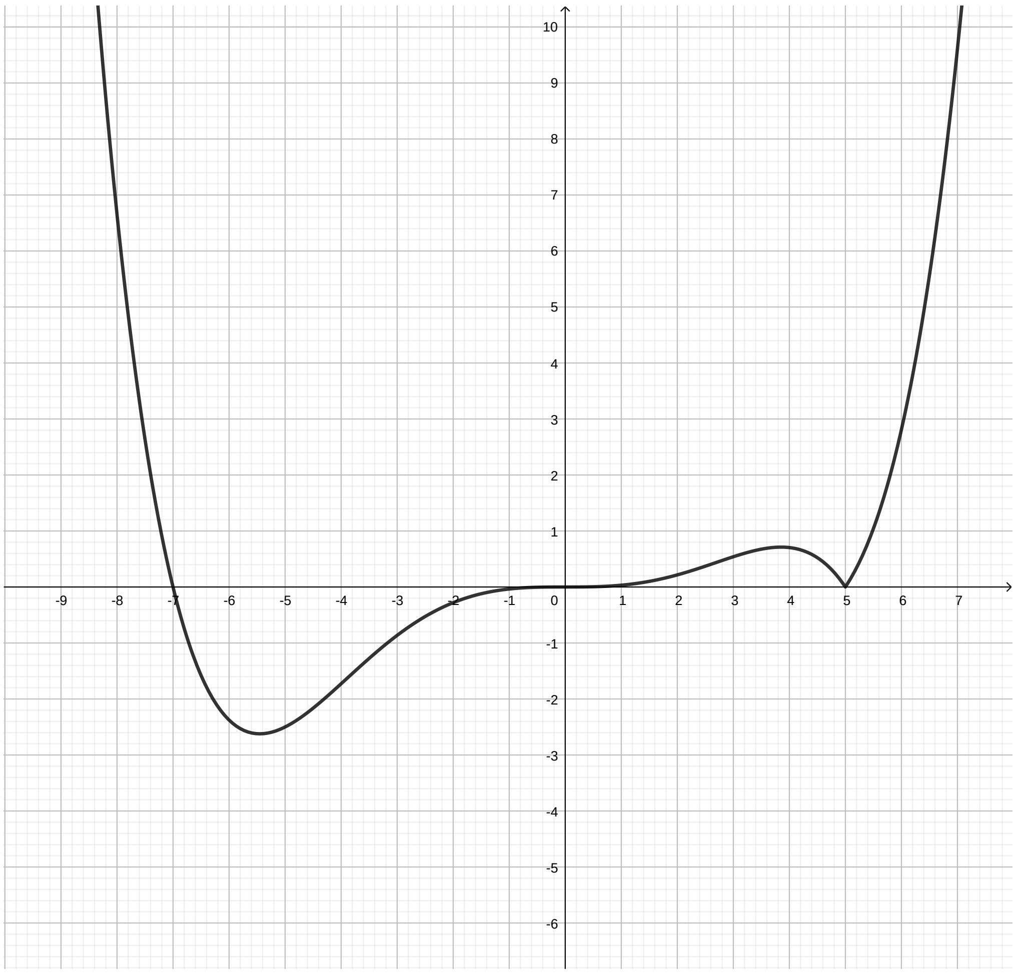
Extrema (4.2)

Find and label the points on the graph of this function that have a tangent line with slope 0. No constraints have been put on the domain, so $dom f = \mathbb{R}$.



Which points are at maximums?

Which points are at minimums?

Which extrema are **local** extremes?

Which extrema are **global** extremes? These are also called **absolute** extremes.

Are there any points at extremes that do not have a tangent line with slope 0?

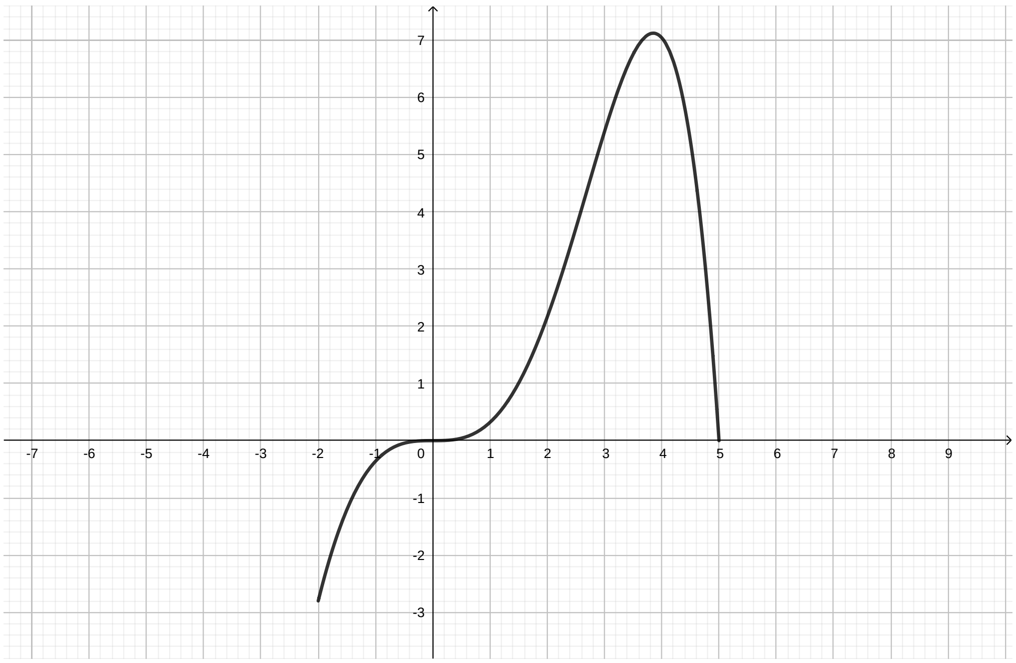
Are there any points with a tangent line with slope 0 that are not extremes?

Definition: A value c in the domain of $f(x)$ is called a **critical point** (c.p.) if $f'(c) = 0$ or $f'(c)$ does not exist.

Find all critical points of $f(x) = x^{-2}\ln(x)$.

Find all critical points of $g(x) = x^{1/3}e^{-x^2}$.

The domain of the function with the following graph is $[-2, 5]$.



Which points are local maximums?

Which points are global maximums?

Which points are local minimums?

Which points are global minimums?

Which points are critical points?

For the following definitions, $a \in I$ where I is an interval.

Definition: $f(a)$ is a **global maximum** of $f(x)$ on I if $f(a) \geq f(x)$ for all $x \in I$. This is also known as an **absolute maximum**.

Definition: $f(a)$ is a **global minimum** of $f(x)$ on I if $f(a) \leq f(x)$ for all $x \in I$. This is also known as an **absolute minimum**.

Definition: $f(a)$ is a **local maximum** of $f(x)$ if there exists an **open** interval[1] I so that $f(a) \geq f(x)$ for all $x \in I$.

Definition: $f(a)$ is a **local minimum** of $f(x)$ if there exists an **open** interval I so that $f(a) \leq f(x)$ for all $x \in I$.

Theorem: If $f(c)$ is an extreme value, then c is a critical point or an endpoint.

Find the absolute extremum of $f(x) = \dfrac{x}{x^2 - x + 1}$ for $x \in [-2, 2]$.

[1] An open interval is of the form (x,y); that is, it does not contain its endpoints.

Section 4.3 starts in 4.2 when the text introduces a special case of the **Mean Value Theorem (MVT)**

If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there exists $c \in (a, b)$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

.

Draw the picture corresponding to this theorem.

Find a number c satisfying the conclusion of the MVT for $h(x) = \dfrac{x}{x+2}$ on the interval $[1, 4]$. Simplify your answer.

The MVT can find an upper bound for the error when using any linear approximation to any function $f(x)$ at $x = a$. The lower bound is 0, no error. Assume $x > a$, then

$$
\begin{aligned}
\text{Error } &= |f(x) - L(x)| \\
&= |f(x) - f(a) - f'(a)(x-a)| \\
&= |f'(c_0)(x-a) - f'(a)(x-a)| \\
&= |f'(c_0) - f'(a)||x-a| \\
&= |f''(c_1)(c_0 - a)||x-a| \\
&< |f''(c_1)(x-a)||x-a| \\
&\leq \max_{a \leq c \leq x} |f''(c)|(x-a)^2.
\end{aligned}
$$

For $x < a$ change the last line to

$$\leq \max_{x \leq c \leq a} |f''(c)|(x-a)^2.$$

(With a more difficult proof we can show the error is $\leq \max_{x \leq c \leq a} \frac{1}{2}|f''(c)|(x-a)^2$ and this is used in Taylor polynomials.)

Estimate $\sqrt{1.2}$ using a linear approximation. Then apply the above formula to our estimate of $\sqrt{1.2}$ to find an upper bound for the error.