L'Hospital's Rule (4.5)

Find the horizontal asymptotes for $f(x) = \dfrac{e^x}{e^x + 1}$.

Now that we have derivatives, we can use a theorem that makes calculating limits with indeterminate forms of $\dfrac{0}{0}$, or $\dfrac{\infty}{\infty}$ easy.

L'Hospital's Rule: If $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ are both 0 or are both $\pm\infty$ then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$ if the limit on the right side exists or is infinite.

Use L'Hospital's Rule to find the horizontal asymptotes for $f(x) = \dfrac{e^x}{e^x + 1}$.

Find the following limits.

1) $\lim_{x \to 0} \frac{1 - \cos(x)}{x^2 + x}$

2) $\lim_{x \to \infty} \frac{e^{0.01x}}{x^2}$.

3) $\lim_{x \to 0^+} x^2 \ln(x)$

4) $\lim_{x \to \pi} \dfrac{\sin(x)}{1 - \cos(x)}$. Careful!

5) $\lim_{x \to \frac{\pi}{2}^-} (\tan(x))^{\cos(x)}$

6) $\lim_{x \to \infty} x^3 e^{-x}$. Generalize.

As time allows:

7) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$.