Applied Optimization (4.7)

Find the point on $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

In the solution, we used the fact that the function $y = \sqrt{x}$ is always increasing to simplify the algebra required to find the minimum. This idea can be generalized into a theorem: The critical points and the corresponding classifications of a composition of functions $(f \circ g)(x)$ where $f(x)$ is always increasing will be the same as those for $g(x)$. Prove this and then give some examples of functions that are always increasing.

A window with a rectangular bottom and semicircular top has perimeter of 10 meters. Find the dimensions of the window that admit the most light. (i.e. maximize area.)

A fence three meters tall runs parallel to a tall building at a distance of two meters from the building. What is the length of the shortest ladder that will reach from the ground outside of the fence to the building?

A cone shaped drinking cup is made to hold 27 cm^3 of water. Find the height and radius of the cup that has the least surface area.