# Newton's Method (4.8)

Finding zeroes of polynomials is important (in your future, they will be eigenvalues!) and it's important to find them quickly (or to program a computer or calculator to do it quickly.) Newton's Method is a fast method that uses derivatives.

The method uses an initial guess to estimate a zero of $f(x)$, call it $g_0$, and then uses the x-intercept of the linear approximation for $f(x)$ at $g_0$ as the first guess, $g_1$. Draw a picture and then prove that

$$g_1 = g_0 - \frac{f(g_0)}{f'(g_0)}.$$

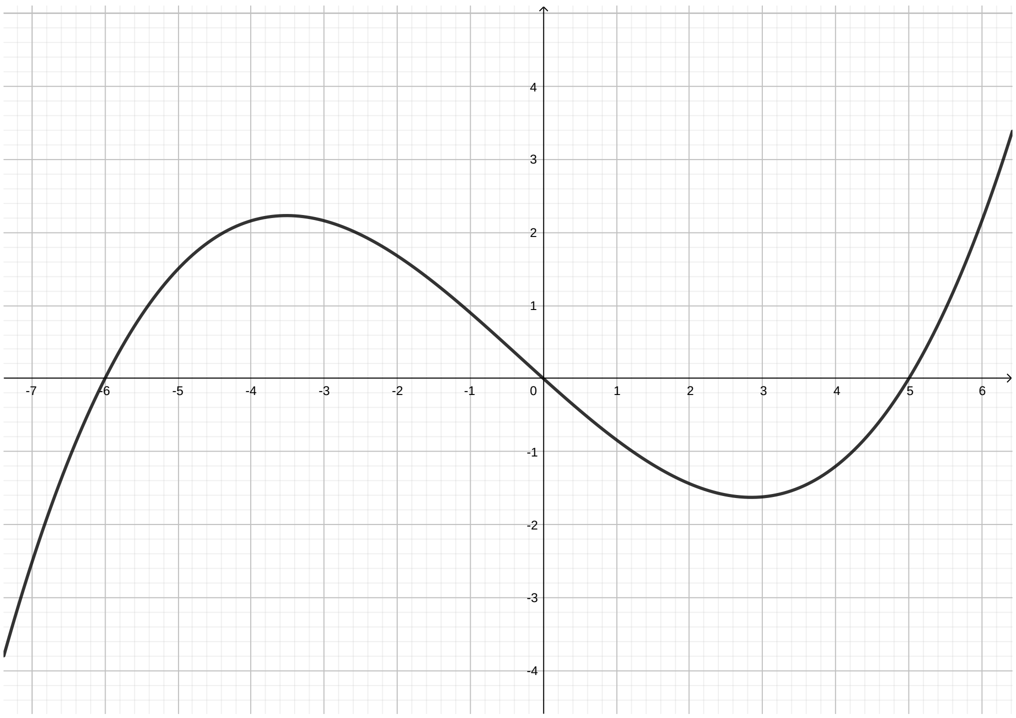
The method then repeats itself and so recursively defines a new guess in terms of the previous one. Draw a picture and then prove that

$$g_{n+1} = g_n - \frac{f(g_n)}{f'(g_n)}.$$

Use what we have learned about shapes of graphs and or the IVT to determine how many zeroes $f(x) = x^3 + x + 1$ has.

Now use Newton's method to approximate the zeroes of $f(x) = x^3 + x + 1$ by calculating $g_3$ using an initial guess of $g_0 = -1$. Make a table to organize your work.

Your initial guess must be somewhat close to the zero to be effective. Find a $g_0$ for the following graph that does not estimate the smaller zero.




Find a Newton's Method table Calculator online to estimate all the roots of $\sin(x) = x^2 - 3x + 1$. We will first need to create a function and determine how many roots it has and approximately where they are located. Use $f(x) = x^2 - 3x + 1 - \sin(x)$ and plot it using a calculator to determine the number of roots and a close guess for each.