Substitution: Reversing the Chain Rule (5.7)

Let $F(u)$ be an antiderivative of $f(u)$. If $u = u(x)$, then the chain rule tells us that $F(u(x))$ is an antiderivative of $f(u(x))u'(x)$. FTC then implies

$$\int_{u(a)}^{u(b)} f(u)\,du = F(u(b)) - F(u(a)) = \int_a^b f(u(x))u'(x)\,dx$$

This equality of the two integrals is conventionally denoted

$$du = \frac{du}{dx}\,dx.$$

The method of substitution looks at the integrand $f(u(x))u'(x)$ and recognizes or guesses what the function $u(x)$ is and then substitutes to get an integral that is easier to integrate: $\displaystyle\int_{u(a)}^{u(b)} f(u)\,du$.

Calculate $\displaystyle\int_{\sqrt{\pi/6}}^{\sqrt{\pi/3}} 2x\cos(x^2)\,dx$ two different ways.

(a) Using the substitution method.

(b) Guessing and checking.

Too often substitution is used when guess and check is faster and less likely to cause errors.

Find $\int \tan(p)\, dp$.

Find $\int_0^1 \frac{x}{3+5x}\, dx$.

Find $\displaystyle\int_{-1}^{2} \sqrt{5x+6}\, dx$.

Find $\displaystyle\int_{0}^{\sqrt{e-1}} \frac{x^3}{x^2+1}\, dx$.

Find $\int_1^3 e^{5x}\,dx$.

Find $\int_0^{\sqrt{\pi/4}} \sec^2(2y - \pi/3)\,dy$