Integration by Parts (7.1)

Integration by parts is the reverse of the product rule. If $u = u(x)$ and $v = v(x)$, then

$$uv' + u'v = (uv)'$$

$$\Longrightarrow\ uv' = (uv)' - u'v$$

$$\Longrightarrow\ \int uv'\,dx = \int (uv)' - u'v\,dx$$

$$\Longrightarrow\ \boxed{\int u\,dv = uv - \int v\,du \qquad (*)}$$

$(*)$ is an identity and should be memorized if you can't derive it yourself from the product rule. There is a nice way to memorize all of the "parts" of this formula: it is called the tabular method.

+	u	dv
-	du	v

Calculate $I = \displaystyle\int_0^{\pi/2} x\cos(x)\,dx$.

The tabular method really helps when integration-by-parts has to be repeated.

Calculate $I = \int_0^1 x^3 e^x \, dx$.

Find $I = \int_1^e \frac{\ln(x)}{x^2} \, dx$.

Calculate $I = \int_{-1}^{1} \frac{\sin(x)}{1 + x^2}\, dx$.

Calculate $I = \int_{0}^{1} \sin^{-1}(x)\, dx$.

Calculate $I = \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \sin(\theta^2)\, d\theta$.

Here is a special example where you must stop using parts and solve an algebraic equation.

Calculate $I = \int e^x \sin(x)\, dx$.

Find $I = \int_0^1 \frac{x^3}{\sqrt{9 + x^2}}\, dx$.

Calculate $I = \int_0^1 \arctan(y)\, dy$.

As time allows:
(a) What is the derivative of $\sinh^{-1}(x)$ with respect to x?

(b) What is the antiderivative of $\sinh^{-1}(x)$?