Final 200 points Math 160 Name: ______________________________

Always show work to defend your answer in a logical and organized fashion.

1. (15 points) B is the region in the first quadrant bounded by $x^2 + y^2 = 4$, $x = 1$, and $y = 0$. Sketch and shade B and then find the volume of the solid generated by rotating B about the y-axis.

2. (10 points) Find the work done pulling a 20 kg weight from the ground to the roof of a 30-meter building using a chain with density 2 kg/meter. Use 10 m/sec^2 for standard gravity.

3. (10 points) Solve $\dfrac{dy}{dx} + \dfrac{2}{1+x}y = 3$ for y explicitly if $y(0) = 2$.

4. (15 points) Solve $\dfrac{dy}{dx} = \dfrac{(y+2)(y+3)}{x}$ for y implicitly if $y(1) = 0$. Recall an **implicit solution** does not need y to be isolated on one side of the equation.

5. (10 points) Find the interval of convergence for $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n n^2}$.

6. (10 points) Compute $I = \displaystyle\int_0^{\pi/4} \tan^2(x)\sec^2(x)\,dx$ using a substitution and changing limits.

7. (15 points) A lamina with constant density is bounded by $x = 4y^2$ and $x = 4$. Sketch a shaded region of the lamina and then find its center of mass.

8. (10 points) Evaluate $S = \sum_{n=2}^{\infty} \frac{(-1)^n(3)^n}{5^n}$ if it converges or prove that it diverges.

9. (10 points) Calculate $I = \int_0^1 \frac{x^2}{\sqrt{4-x^2}}\,dx$ using a trigonometric substitution.

10. (15 points) Compute $I = \int_0^\infty x^2 e^{1-x}\,dx$ or prove that it diverges.

11. (10 points) Find the length of the curve parameterized by $x(t) = 2t$, $y(t) = 2\cosh(t)$ from $t = 0$ to $t = \ln(10)$. Recall $\cosh^2(t) - \sinh^2(t) = 1$.

12. (10 points) Does $S = \sum_{n=1}^{\infty} \frac{n^2 - 1}{\sqrt{n^5 + 1}}$ converge or diverge? Defend your answer.

13. (15 points) Find the area in the xy-plane inside the cardioid $r = 1 + \cos(\theta)$ but outside the circle $r = 1$. Sketch and shade the area.

14. Use the points $P = (1, 2, 2)$, $Q = (2, -1, 2)$, and $R = (1, 1, 1)$ to answer the following.

   (a) (10 points) Find an equation for the plane in standard form that contains the points P, Q, and R.

   (b) (10 points) Find $\overrightarrow{PQ}_{||\overrightarrow{PR}}$, the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$.

15. (15 points) Find the first two nonzero terms for the Taylor series of $f(x) = \arctan(x)$ expanded about $x = 0$, and then estimate $I = \int_0^1 \arctan(x^2)\, dx$ using those nonzero terms and a substitution. Recall that $f'(x) = \dfrac{1}{1+x^2}$ and please observe the input of the arctan in the integrand is x^2, not x.

16. (10 points) $\vec{u} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{v} =< 2, -1, -2 >$. Simplify

$$3\vec{u} \cdot \vec{v} + ||\vec{v}|| - (\vec{v} + \vec{u}) \cdot \hat{\jmath}$$