Final 100 points Math 160 Name: ______________________________

Always show organized work.

1. (6 points) B is the region in the first quadrant bounded by $x + y = 4$, $x = 1$, and $y = 0$. Find the volume of the solid generated by rotating B about the y-axis.

2. (6 points) Find the work done pumping ice cream out of the top of a giant conical ice cream cone that has a height of 2 meters and radius of 4 meters. Use ρ for the density of ice cream and g for standard gravity.

3. (6 points) A half-disk with constant density is bounded by $x^2 + y^2 = 4$ for $x \geq 0$. Find the center of mass.

4. (9 points) Find the interval of convergence for $f(x) = \sum_{n=1}^{\infty} \frac{n(x-2)^n}{3^n}$.

5. (6 points) Solve $\frac{dy}{dx} + \left(\frac{1}{x} - 1\right) y = \frac{2e^x \cos(x) \sin(x)}{x}$ for y explicitly if $y(\pi) = 0$.

6. (4 points) Evaluate $S = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n)!}$ if it converges or prove that it diverges.

7. (6 points) Find the length of the curve parameterized by $x(t) = 3t^2$, $y(t) = 2t^3$ from $t = 0$ to $t = 2$.

8. (6 points) Find the area in the xy-plane inside the closed curve $r = 2 + \cos(2\theta)$.

9. (9 points) Find the first two nonzero terms for the Taylor series of $f(x) = \sec(x)$ expanded about $x = 0$, and then estimate $I = \int_0^1 \sec(x^2)\,dx$ using those nonzero terms and a substitution.

10. (6 points) Solve $\frac{dy}{dx} = \frac{y}{(x+2)(x+3)}$ for y explicitly if $y(0) = 1$.

11. (6 points) Compute $I = \int_{-\pi}^{\pi} (\sin(x) + 2\cos(x))^2 \, dx$.

12. (6 points) Compute $I = \int_{0}^{\infty} x^2 e^{1-x} \, dx$.

13. (6 points) Calculate $I = \int_{\frac{2}{\sqrt{3}}}^{2} \frac{dx}{x\sqrt{x^2-1}}$.

14. Use the points $P = (1, -2, 1)$, $Q = (3, 2, 1)$, and $R = (1, 2, 3)$ to answer the following.

(a) (4 points) Find $\overrightarrow{PQ}_{||\overrightarrow{PR}}$, the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$.

(b) (4 points) Find an equation for the plane in standard form that contains the points P, Q, and R.

15. (4 points) Simplify $3\vec{u} \cdot \vec{v} + ||\vec{v}|| - (\vec{v} + \vec{u}) \cdot \hat{\jmath}$ if $\vec{u} = 3\hat{\imath} - \hat{\jmath}$ and $\vec{v} =< 0, 3, 4 >$.

16. (6 points) Does $S = \sum_{n=1}^{\infty} \frac{\sqrt{n^5 + 1}}{n^4}$ converge or diverge? Defend your answer.