Final 200 points Math 160 Name: ______________________________

Always show work to defend your answer in a logical and organized fashion.

1. (15 points) B is the region in the first quadrant bounded by $y = x(1 - x)$ and the x-axis. Sketch and shade B and then find the volume of the solid generated by rotating B about the y-axis.

2. (10 points) Find the work done pulling a 30 kg weight from the ground to the roof of a 20-meter building using a chain with density 6 kg/meter. Use 10 m/sec^2 for standard gravity.

3. (10 points) Solve $\dfrac{dy}{dx} + \dfrac{2}{x}y = \cos(x)$ for y explicitly if $y(\pi) = 0$.

4. (10 points) Solve $\sqrt{1-x^2}\,\dfrac{dy}{dx} = x(y-1)(y-2)$ for y implicitly if $y(0) = 0$.

5. (15 points) Find the interval of convergence for $f(x) = \displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{(x-2)^n}{6^n \ln(n)}$.

6. (10 points) Compute $I = \displaystyle\int_0^{\pi} (\sin(9x) + \cos(x))^2\, dx$.

7. (15 points) A lamina with constant density is bounded by $y = x^2$, $y = -x^2$ and $x = 2$. Sketch a shaded region of the lamina and then find its center of mass.

8. (10 points) Evaluate $S = \displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$ if it converges or prove that it diverges. Notice the series starts at $n = 2$.

9. (10 points) Calculate $I = \displaystyle\int_0^{\sqrt{3}/2} \frac{x^3}{\sqrt{1-x^2}}\,dx$ by first using a trigonometric substitution.

10. (15 points) Find the length of the curve parameterized by $x(t) = \frac{4}{3}t^{3/2} + 1$, $y(t) = \frac{1}{2}(t-1)^2 + 3$ from $t = 2$ to $t = 4$.

11. (10 points) Compute $I = \int_0^\infty 2e^{-kt}\,dt$ in terms of the constant k. For what values of k does I converge? Defend your answer.

12. (10 points) Find the values of t in the interval $[0, 2\pi]$ for which the trace of $\vec{p}(t) = \langle e^t\cos(t), e^t\sin(t)\rangle$ has a horizontal tangent line in the xy - plane.

13. (15 points) Find the area in the xy-plane inside the four leaf clover $r = 2\cos(2\theta)$ but outside the circle $r = 1$. Sketch and shade the area.

14. Use the points $P = (1, 1, 0)$, $Q = (1, 0, 1)$, and $R = (0, 1, 1)$ to answer the following.

(a) (10 points) Find an equation for the plane in standard form that contains the points P, Q, and R.

(b) (10 points) Find $\overrightarrow{PQ}_{||\overrightarrow{PR}}$, the projection of $\overrightarrow{PQ}$ onto $\overrightarrow{PR}$.

15. (15 points) Find the first three nonzero terms for the Taylor series of $f(x) = e^{\sin(x)}$ expanded about $a = 0$ using differentiation, and then estimate $I = \int_0^1 e^{\sin(x)}\,dx$ using those nonzero terms.

16. (10 points) $\vec{u} = 2\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and $\vec{w} = \langle -1, 4, 8 \rangle$. Simplify $\hat{\imath} \cdot \vec{u} \|\vec{w}\| + (\vec{w} \times \vec{u}) \cdot \langle 1, -2, 1 \rangle$.