25 points Math 160 Homework 1 Sections 5.7, 6.1 to 6.4 23 problems

1. What is $\displaystyle\int_{-2}^{2} \frac{x^5\sqrt{4-x^2}}{\cosh^4(x)}\,dx$ and why? Spend less than one minute attempting to solve this problem.

2. Evaluate $\displaystyle\int_{0}^{\pi/3} \frac{\sin(\theta)}{\cos^4(\theta)}\,d\theta$

3. Evaluate $\displaystyle\int_{0.5}^{0.5e^{\pi/4}} \frac{\cos(\ln(2x))}{x}\,dx$

4. Sketch the region enclosed by $y = \tan(x)$ and $y = 2\sin(x)$ for $-\pi/3 \le x \le \pi/3$ and then find the area of the region by integrating with respect to either x or y. Draw a typical approximating rectangle labeling its height and width.

5. Sketch the region enclosed by $x = 2y$ and $x + 1 = (y-1)^2$ and then find the area of the region by integrating with respect to either x or y. Draw a typical approximating rectangle labeling its height and width.

6. Find the **net area** under $y = \sin(x)$ from $0 \le x \le 2\pi$ and then find the **area** bounded by $y = \sin(x)$ and $y = 0$ over the same interval.

7. Find the volume of the solid with base $|x| + |y| = 1$ and semicircular vertical cross sections perpendicular to the y-axis with diameter along the base.

8. A concave-up hemispherical basin of radius R meters holds water. To what percent of capacity is it filled when the water is $0.5R$ meters deep?

9. A six centimeter long strip of mineral deposit has a density of $\rho(x) = 0.01x(6-x)$ grams per centimeter, $0 \le x \le 6$. Calculate the total mass of the deposit.

10. The linear density of a rod 8 meters long is $\rho(x) = \dfrac{12}{\sqrt{x+1}}$ kilograms per meter, $0 \le x \le 8$. Find the mass of the rod.

11. The population density of rabbits r meters away from the center of Ellwood Shores park is $\rho(r) = \dfrac{10}{100^2 + r^2}$ rabbits per square meter. How many rabbits are within 100 meters from the center?

12. Find the average value of $f(x) = e^{kx}, 0 \le x \le 2$.

13. The average temperature in Celsius t hours after 9 a.m. in April at SBCC is modeled by $T(t) = 16 + 7\sin(\pi t/12)$. Find the average of $T(t)$ from 8 a.m. to 5 p.m.

R is the region between $y = e^x$, $x = 0$, $x = 1$, and $y = 0$.

14. Find the volume of the solid formed by rotating region R about the x-axis.

15. Find the volume of the solid formed by rotating region R about the y-axis. Hint: If you can't guess an anti-derivative, then use integration by parts.

K is the region between $f(x) = \sqrt{x}$, $g(x) = x^2$, $x = 0$, and $x = 1$.

16. Find the volume of the solid formed by rotating region K about the x-axis.
17. Find the volume of the solid formed by rotating region K about the line $y = 2$.
18. Find the volume of the solid formed by rotating region K about the line $x = -1$ in two different ways, using both shells and washers.

P is the region between $y = \dfrac{1}{1+x^2}$, $y = 0$, $x = 0$, and $x = 3$.

19. Find the volume of the solid formed by rotating region P about the y-axis.
20. Find the volume of the solid formed by rotating region P about the line $x = 4$.

21. Find the volume of the solid formed by revolving the region bounded by the y-axis, $y = \dfrac{1}{m}x$, $y = a$, and $y = b$ about the y-axis for $0 < a \leq y \leq b$, and $m > 0$.
22. Find the volume of the intersection of two balls, each with radius R, if the center of each ball lies on the others' bounding sphere.
23. CAS Problem (3 points): Turn in a printed copy of the commands and answers.

    A) Evaluate $\displaystyle\int \frac{dy}{e^y(3e^y+2)}$ and $\dfrac{d[2^x \int_0^{x^2} \sin(5p^2)\,dp]}{dx}$. Use symbolic symbols if using Matlab.

    B) Estimate $\displaystyle\int_0^2 \cos(t^2)dt$ to four significant digits using the definition of the definite integral with 100 sub-intervals and the right endpoint of each sub-interval. Use a numeric variable if using Matlab.

Answers

1. 0
2. 7/3
3. $\sqrt{2}/2$
4. $2 - \ln(4)$
5. 32/3
6. 0 and 4
7. $\pi/3$
8. 31.25%
9. 0.36 g
10. 48 kg
11. $10\pi \ln(2)$ rabbits
12. $\dfrac{e^{2k}-1}{2k}$
13. $\approx 20.36°\text{C}$
14. $\dfrac{\pi}{2}(e^2-1)$
15. 2π
16. 0.3π
17. $\dfrac{31}{30}\pi$
18. $\dfrac{29\pi}{30}$
19. $\pi \ln(10)$
20. $\pi[8\arctan(3) - \ln(10)]$
21. $\dfrac{m^2\pi}{3}(b^3-a^3)$
22. $\dfrac{5}{12}\pi R^3$