25 points Math 160 Homework 2 Sections 6.5, 7.1 to 7.3 23 problems

Use $g = 10$ m/sec^2 for standard gravity and $\rho = 1000$ kg/m^3 for the density of water.

1. Find the work done by winding up a hanging 100 m cable with mass-density five kg/m if a 20 kg mass is attached to the end of it.

2. A spring has a natural length of 20 cm. A 25 Newton force is required to keep it stretched at a length of 30 cm. How much work is required to stretch it from 20 cm to 25 cm?

3. A five kg bucket is lifted from the ground to the top of a building that is 10 meters high at a constant speed using a rope with density 0.5 kg/m. Initially the bucket contains 36 kg of water that leaks out at a constant rate so that only six kg of water is left when the bucket is 10 m high. How much work was done?

4. Find the work done pumping water out from a full concave-up 10-meter radius hemispherical tank through a spout on top that is 2 meters high.

5. Find the amount of work required to pump water out of a full horizontal cylindrical tank through a hole on top. Use L for the length and R for the radius of the tank.

Evaluate the integrals using techniques discussed in class.

6. $\displaystyle\int x^2 \sin(x)\, dx$

7. $\displaystyle\int \sin^4(y)\, dy$

8. $\displaystyle\int \frac{x^2}{\sqrt{9-x^2}}\, dx$

9. $\displaystyle\int \frac{\cos^5(x)}{\sin^5(x)}\, dx$

10. $\displaystyle\int_0^{\pi/3} \tan(m)\, dm$

11. $\displaystyle\int \frac{1}{\sqrt{x^2+4x+13}}\, dx$

12. $\displaystyle\int e^x \cos(x)\, dx$

13. $\displaystyle\int \sin^3(t)\cos^2(t)\, dt$

14. $\displaystyle\int \cos(5x)\cos(3x)\, dx$

15. $\displaystyle\int_0^{2\pi} \sin^2(\theta)\, d\theta$

16. $\displaystyle\int_0^3 \frac{dt}{(t^2+9)^2}$

17. $\displaystyle\int (\ln(x))^2\, dx$

18. $\displaystyle\int \tan^2(p)\sec^4(p)\, dp$

19. $\displaystyle\int \ln(x^2+1)\, dx$

20. $\displaystyle\int_0^{\pi/2} \sin(6x)\cos(3x)\, dx$

21. Find the average height of a point on the semicircle $y = \sqrt{1 - x^2}$, $-1 \le x \le 1$.

22. A spring requires six Joules of work to stretch from 10 cm (or .10 m) to 12 cm and an additional 10 Joules to stretch from 12 cm to 14 cm. What is the natural length of the spring? Hint: Divide two equations and cancel the spring constant.

23. CAS Problem (3 points): Use a CAS to solve the following. Turn in a printed copy of the commands and answers.

    Find $\int (1 + \ln(x))\sqrt{1 + x^2 \ln^2(x)}\, dx$. Note: Matlab and many other CAS's use $\log(x)$ instead of $\ln(x)$.

Answers

1. $27 \cdot 10^4$ J
2. 0.3125 J $= \dfrac{5}{16}$ J
3. 2850 J
4. $\dfrac{115}{3} \cdot 10^6 \pi$ J
5. $L\pi R^3 \cdot 10^4$ J
6. $-x^2 \cos(x) + 2x \sin(x) + 2\cos(x) + C$
7. $\dfrac{3y}{8} - \dfrac{\sin(2y)}{4} + \dfrac{\sin(4y)}{32} + C$
8. $\dfrac{9}{2}\arcsin(x/3) - \dfrac{x}{2}\sqrt{9 - x^2} + C$
9. $-\dfrac{1}{4\sin^4(x)} + \dfrac{1}{\sin^2(x)} + \ln(\sin(x)) + C$
10. $\ln(2)$
11. $\ln\left|\sqrt{1 + \dfrac{(x+2)^2}{9}} + \dfrac{x+2}{3}\right| + C$
12. $\dfrac{e^x}{2}(\cos(x) + \sin(x)) + C$
13. $\dfrac{\cos^5(t)}{5} - \dfrac{\cos^3(t)}{3} + C$
14. $\dfrac{\sin(8x)}{16} + \dfrac{\sin(2x)}{4} + C$
15. π
16. $\dfrac{\pi + 2}{216}$
17. $x\ln^2(x) - 2x\ln(x) + 2x + C$
18. $\dfrac{\tan^3(p)}{3} + \dfrac{\tan^5(p)}{5} + C$
19. $x\ln(x^2 + 1) - 2x + 2\arctan(x) + C$
20. $\dfrac{2}{9}$
21. $\pi/4$
22. 8 cm