

Evaluate the following integrals.

1. $\int x \sinh(x^2 + 2) dx$

6. $\int \frac{p^2 + 11p}{(p-1)(p+1)^2} dp$

2. $\int_{-1}^1 \tanh(x) dx$

7. $\int \frac{3p^2 - 4p + 5}{(p+1)(p^2+1)} dp$

3. $\int \frac{1}{\sqrt{4x^2-9}} dx$

8. $\int \frac{6x^2 + 2}{x^2 + 2x - 3} dx$

4. $\int \frac{1}{(y-2)(y-4)} dy$

9. $\int_{-\pi}^{\pi} (\sin(\theta) + \cos(2\theta))^2 d\theta$

5. $\int_{-1}^0 \frac{y^2 + 3y - 44}{(y+3)(y+5)(3y-2)} dy$

10. $\int x^2 \ln(x) dx$

11. Which are improper integrals? Defend your answer.

(a) $\int_{-1}^{\infty} e^{-x} dx$

(b) $\int_0^1 e^{-x} dx$

(c) $\int_0^{\pi} \sec(x) dx$

(d) $\int_0^1 \frac{dx}{\sqrt{3-x^2}}$

Evaluate if the integral is convergent or show why it is divergent.

12. $\int_2^{\infty} x^{-3} dx$ vs. $\int_0^2 x^{-3} dx$

15. $\int_1^{\infty} \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

13. $\int_{-1}^1 x^{-0.2} dx$

16. $\int_{-\infty}^{\infty} \frac{x}{1+x^2} dx$

14. $\int_0^{\infty} e^{-x} \cos(x) dx$

17. $\int_{-\infty}^{\infty} x dx$ vs. $\lim_{R \rightarrow \infty} \int_{-R}^R x dx$

Use the comparison test to determine if the integral converges or diverges.

18. $\int_1^{\infty} \frac{1 - \sin(x)}{x^2} dx$

20. $\int_0^{\infty} \frac{1}{x + e^x} dx$

19. $\int_0^1 \frac{e^x}{x^2} dx$

21. $\int_5^{\infty} \frac{1}{x^p \ln(x)} dx$ for any real number p .

22. Find the constant C that makes $p(x) = \frac{C}{\sqrt{1-x^2}}$ on $(-1, 1)$ a probability distribution function. Then find the probability that a random variable with this distribution is between $-1/2$ and $1/2$.
23. CAS Problem (3 points): Use a CAS to compute $\int_0^\infty x^5 e^{-x^2} dx$. Submit a printed copy of the commands and answers.

Answers

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|--|---|
| 1. $\frac{\cosh(x^2 + 2)}{2} + C$ | 10. $\frac{x^3 \ln(x)}{3} - \frac{x^3}{9} + C$ |
| 2. 0 | 11. (a) and (c) are improper. |
| 3. $\pm \frac{1}{2} \ln \left \frac{2x \pm \sqrt{4x^2 - 9}}{3} \right + C$ | 12. 1/8 vs. divergent |
| 4. $\frac{1}{2} \ln \left \frac{y-4}{y-2} \right + C$ | 13. 0 |
| 5. $\ln \left(\frac{9}{20^{1/3}} \right)$ | 14. 1/2 |
| 6. $\ln \left \frac{(p-1)^3}{(p+1)^2} \right - \frac{5}{p+1} + C$ | 15. diverges |
| 7. $6 \ln p+1 - \frac{3}{2} \ln(p^2+1) - \arctan(p) + C$ | 16. diverges |
| 8. $6x + \ln \left[\frac{(x-1)^2}{(x+3)^{14}} \right] + C$ | 17. diverges vs. 0 |
| 9. 2π | 18. converges |
| | 19. diverges |
| | 20. converges |
| | 21. diverges if $p \leq 1$ and converges if $p > 1$. |
| | 22. $C = 1/\pi$ and 1/3 probability. |