25 points Math 160 Homework 4 Section 7.8, 8.2 - 8.4 23 problems

Let $I = \int_0^3 \sin^2(\pi x)\,dx$ **for # 1 - 6.**

1. Calculate I.
2. Estimate I using the Midpoint method with n = 6.
3. Estimate I using the Trapezoid method with n = 6.
4. Estimate I using Simpson's method with n = 6.
5. Show that each estimate is within the prescribed bounds dictated by the error formulas in your book.
6. What is the least n required to guarantee the error for estimating I is smaller than .001 using Simpson's method?

7. Calculate the arc length of $y = \dfrac{x^2}{4} - \dfrac{\ln(x)}{2}$ over the interval $[1, e^2]$.
8. Find the value of a so that the arc length of $y = \cosh(x)$ over the interval $[-a, a]$ equals 10.
9. Calculate the arc length of $y = \ln(\cos(x))$ over the interval $[0, \pi/4]$.
10. Compute the area of the surface of revolution of $y = x^3$ about the x-axis over the interval $[0, 2]$.
11. Compute the area of the surface of revolution of $y = \sqrt{1 + 4x}$ about the x-axis over the interval $[1, 5]$.
12. Compute the area of the surface of revolution of $y = (4 - x^{2/3})^{3/2}$ about the x-axis over the interval $[0, 8]$.
13. Compute the surface area of the torus formed by rotating $(x - 2)^2 + y^2 = 1$ about the y-axis.

Use $\rho = 1000$ kg/m^3 for the density of water, and $g = 10$ m/sec^2 for the acceleration due to gravity.

14. The vertical ends of a water trough are isosceles right triangles with legs 3 meters long and the 90 degree angle at the bottom. Calculate the force on one end when the trough is full of water.
15. Each end of a horizontal oil tank is an ellipse with horizontal axis 4 meters and vertical axis 2 meters. Calculate the force on one end when the tank is **half** full of oil with density 800 kg/m^3.
16. A 6 meter wide rectangular boat ramp inclined at 30 degrees rises from beneath the water 4 meters vertically before reaching the surface. Calculate the force of the water on the ramp.

17. Find the center of mass for the one-dimensional metal rod with linear density $\delta(x) = x^2$ for $0 \leq x \leq 1$.

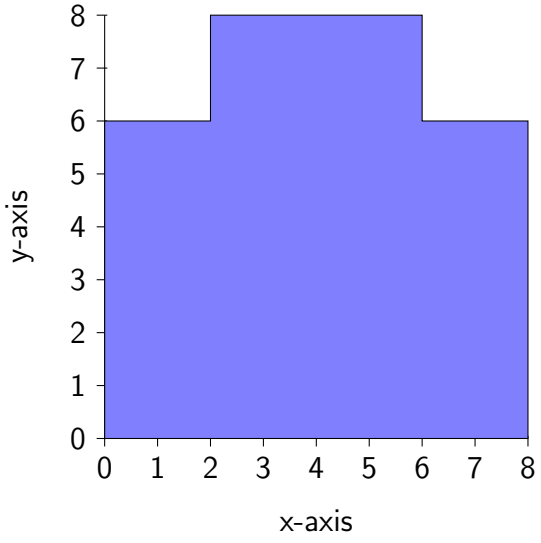
Find the centroid for the following regions.

18. The region between the graph of $y = \sqrt{x}$, the x-axis, and $x = 4$.

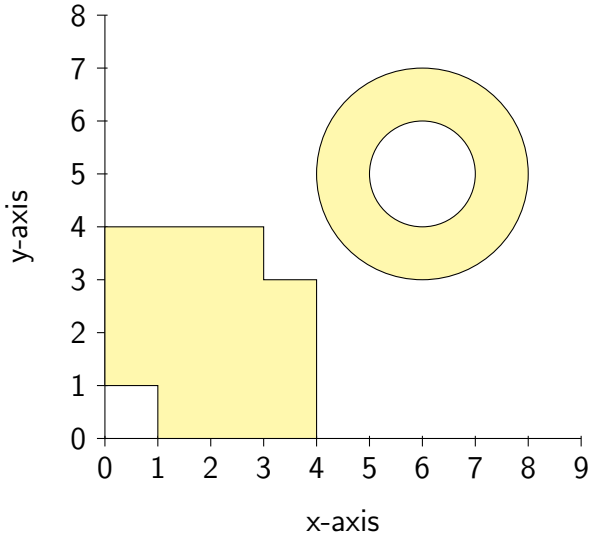
19. The region between $y = 1$ and $y = e^x$ for $0 \leq x \leq 1$.

20. The region inside $x^2 + y^2 = 16$ to the right of $x = 2\sqrt{3}$.

21.



22.



23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of your commands and answer. Use Simpson's method to estimate $\int_0^1 \cosh(x^2)\,dx$ with an error less than 10^{-8}. Use the minimal number of subintervals to do this - what is that number?

Answers

1. 1.5
2. 1.5
3. 1.5
4. 2
5. Simpson's error bound is 0.812.
6. n=34
7. $0.25(e^4 + 3)$
8. $a = \ln(\sqrt{26} + 5)$
9. $\ln(\sqrt{2} + 1)$
10. $\frac{\pi}{27}(145^{3/2} - 1)$
11. $\frac{98\pi}{3}$
12. $\frac{384\pi}{5}$
13. $8\pi^2$
14. $\frac{9 \cdot 10^4}{2\sqrt{2}}$ Newtons
15. $\frac{32000}{3}$ Newtons
16. $96 \cdot 10^4$ Newtons
17. $\frac{3}{4}$
18. $\left(\frac{12}{5}, \frac{3}{4}\right)$
19. $\left(\frac{1}{2e-4}, \frac{e^2-3}{4e-8}\right)$
20. $\left(\frac{4}{2\pi - 3\sqrt{3}}, 0\right)$
21. $\left(4, \frac{25}{7}\right)$
22. $\left(\frac{18\pi + 28}{3\pi + 14}, \frac{15\pi + 28}{3\pi + 14}\right)$