25 points Math 160 Homework 5 Chapter 9 23 problems

1. Solve $y' = 4y + 24, y(0) = 5$ by inspection using only a tiny bit of algebra.

2. Police find a dead body at 3 a.m. and measure its temperature to be 27°C (normal temperature is 37°C.) One hour later the body's temperature is 25°C. Determine the time of death assuming the body was found in a room with constant temperature of 20°C.

3. A metal bar with temperature −30°C is submerged in a pool maintained at 30°C. 30 seconds later the bar's temperature is 10°C. How long after being placed in the pool will the bar have a temperature of 20°C?

4. What is the terminal velocity of a 60 kg skydiver in meters/sec? use $v' = \dfrac{-10v}{m} - g$ and write your answer in terms of the standard gravity g.

5. Suppose you borrow $ 20,000 at a 5% interest rate compounded continuously to buy a car. If you are to pay off the loan in 5 years, how many dollars, N, must you pay each year? Solve $y' = 0.05y - N$ to find N if y represents the amount you owe at time t.

Use separation of variables to find the solution to each equation.

6. $\dfrac{dy}{dt} - 20t^4 e^{-y} = 0$

7. $\sqrt{1-x^2}y' = xy$

8. $\dfrac{dx}{dt} = (t+1)(x^2+1)$

9. $yy' = xe^{-y^2}, y(0) = -2$

10. $t^2\dfrac{dy}{dt} - t = 1 + y + ty$

11. $\dfrac{dy}{dt} = 3y\left(1 - \dfrac{y}{5}\right),\ y(0) = 4$

12. The ground squirrel population living near a farm has a carrying capacity of 500 squirrels. Assume logistic growth with constant $k = 0.5\ \text{yr}^{-1}$ if the farmer does not trap them. If P(0)=10, how long before the population is 100 squirrels?

Solve the following linear differential equations.

13. $y' + 3x^2 y = x^2$

14. $x\dfrac{dy}{dx} - y = x^2\sin(x)$

15. $\cos(x)\dfrac{dy}{dx} + \sin(x)y = 1$

16. $\dfrac{dp}{dt} + 2tp = p + 4t - 2$

17. $y' + 4xy = x^3 e^{x^2}, y(0) = -1$

18. $y\dfrac{dx}{dy} - x = 2y^2, x(5) = 1$

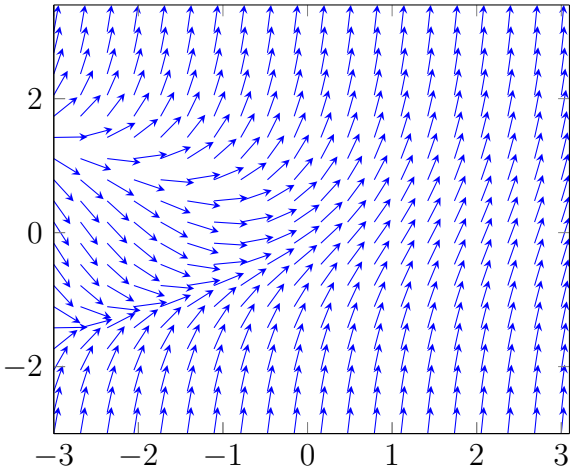
19. Water flows into a tank at $\dfrac{30}{1+t}$ liters/min and out at 20 liters/min. Solve for $v(t)$ =volume of water in the tank at time t if $v(0) = 400$ liters.

20. Water polluted with 5 grams of toxin per cubic meter empties into a lake at 1000 cubic meters per hour. The lake has 200,000 cubic meters of water and water leaves the lake at 1000 cubic meters per hour. We assume the toxin entering the lake is instantaneously and evenly mixed into the water of the lake and that no toxin is initially in the lake. If the pollution is unchecked, how many grams of toxin will eventually be in the lake?

21. Sketch the slope field for $\dfrac{dy}{dx} = y - x^2$, $-2 \le x \le 2$, by hand (without using a computing device) using several isoclines.

22. Which equation corresponds to the given slope field? Defend your answer.

(a) $y' = 1 + y^2$ (b) $y' = 1 + y^2 + x$ (c) $y' = 1 - y^2 + x$

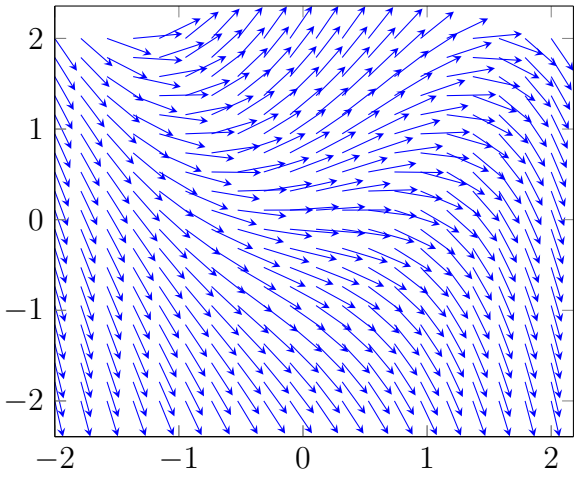


23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of the commands and answers.

Find the exact solution for the IVP $y' = 100\sin(x) - 100y$, $y(0) = 0$. Then using this answer, approximate $y(3)$ to four digits.

Answers

1. $y = 11e^{4t} - 6$
2. About 12:22 a.m.
3. $\dfrac{\ln(6)}{\ln(9)}$ minutes $\approx$ 49 seconds
4. $-6g$ m/sec
5. $N = \dfrac{1000}{1 - e^{-.25}} \approx \$4520.81$/year
6. $y = \ln(4t^5 + C)$
7. $y = \pm e^{C - \sqrt{1-x^2}}$
8. $x = \tan\left(\dfrac{t^2}{2} + t + C\right)$
9. $y = -\sqrt{\ln(x^2 + e^4)}$
10. $y = |t|e^{C - t^{-1}} - 1$
11. $y = \dfrac{20}{e^{-3t} + 4}$
12. ≈ 5.01 years
13. $y = \dfrac{1}{3} + Ce^{-x^3}$
14. $y = -x\cos(x) + Cx$
15. $y = \sin(x) + C\cos(x)$
16. $p = 2 + Ce^{t - t^2}$
17. $\dfrac{1}{18}\left(e^{x^2}(3x^2 - 1) - 17e^{-2x^2}\right)$
18. $x = 2y^2 - \dfrac{49y}{5}$
19. $v(t) = 30\ln(1 + t) - 20t + 400$
20. 10^6 grams of toxin
21. isoclines are $y = x^2 + k$.



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