

For # 1 - 5, list four terms of each sequence starting with $n = 0$ if possible or $n = 1$ otherwise. Then determine the limit of the sequence or prove divergence.

1. $a_n = \cos^{-1}\left(\frac{n^3}{2n^3 + 1}\right).$

3. $c_n = \frac{e^n}{2^n}$

2. $b_n = 10 + \left(-\frac{1}{9}\right)^n$

4. $p_n = \left(1 + \frac{1}{n}\right)^n$

5. $w_n = (2^n + 3^n)^{1/n}$ Hint: squeeze.

For # 6 - 9, find the sum of the following series or prove divergence.

6. $\sum_{n=-2}^{\infty} (3/11)^n$

8. $\sum_{n=0}^{\infty} \frac{3 + 2^n}{5^n}$

7. $\sum_{n=1}^{\infty} \cos(1/n)$

9. $\sum_{n=5}^{\infty} (2/3)^{-n}$

10. Determine a reduced fraction x with repeating decimal 0.4545... by writing x as a geometric series.

11. $S = \sum_{n=1}^{\infty} a_n$ has partial sum $S_k = 2 - 5e^{-k}$. Find the following.

(a) S

(b) $\sum_{n=5}^{16} a_n$

(c) a_n

For # 12 - 19, determine convergence or divergence. Defend your answer by naming the test used and verifying that test's hypotheses.

12. $\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^2}$

16. $\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^3 + 1}}$

13. $\sum_{k=1}^{\infty} \frac{\sin^2(k)}{k^2}$

17. $\sum_{n=2}^{\infty} \frac{1}{n^{1/2} \ln(n)}$

14. $\sum_{n=0}^{\infty} 2^{-n^2}$

18. $\sum_{k=1}^{\infty} \frac{1}{5^k - 3^k}$

15. $\sum_{k=1}^{\infty} \frac{3^k}{k}$

19. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{1/2}(\ln(n))^2}$

For # 20 to 22, determine if each series converges absolutely, converges conditionally, or diverges. Defend your answer.

20. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2}$

21. $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{1.1^n}$

22. $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{(\ln(n))^2}$

23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of the commands and answers. The alternating harmonic series, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges to $\ln(2)$. A) Sum the first 100 terms and subtract from $\ln(2)$ to show the error is smaller than 0.01. B) Sum the first 1000 terms. How accurate is this estimate of $\ln(2)$?

Answers

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|---|-----------------------------|
| 1. $\pi/3$ | 12. converges |
| 2. 11, 89/9, 811/81, 7289/729; limit = 10 | 13. converges |
| 3. 1, $e/2$, $e^2/4$, $e^3/8$; c_n diverges. | 14. converges |
| 4. 2, 2.25, 2.37, 2.44; limit = e | 15. diverges |
| 5. 5, 3.61, 3.27, 3.14; limit = 3 | 16. diverges |
| 6. $\frac{1331}{72} \approx 18.49$ | 17. diverges |
| 7. diverges | 18. converges |
| 8. 65/12 | 19. converges |
| 9. diverges | 20. converges absolutely |
| 10. 45/99 | 21. converges absolutely |
| 11. a) 2 b) $5e^{-4} - 5e^{-16}$ c) $5e^{1-n} - 5e^{-n}$ | 22. converges conditionally |