25 points Math 160 Homework 7 Sections 10.5 to 10.8 23 problems

1. Use the ratio test to determine the behavior of $\sum_{n=1}^{\infty} \frac{n^2}{(2n+1)!}$.

2. Use the root test to determine the behavior of $\sum_{k=0}^{\infty} \left(\frac{3k+1}{k+1}\right)^k$.

Find the interval of convergence. Defend your answer.

3. $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

4. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$

5. $\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$

6. $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{n2^n}$

7. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{\sqrt{n^2+1}}$

8. $\sum_{n=2}^{\infty} \frac{x^n}{\ln(n)}$

9. $\sum_{n=1}^{\infty} (-1)^n n^5 (x-7)^n$

10. $\sum_{n=0}^{\infty} \frac{(-5)^n (x+10)^n}{n!}$

11. Find the Taylor series for each function about $x = 0$.

    (a) e^x

    (b) $\cos(x)$

    (c) $\sin(x)$

    (d) $\frac{1}{1-x}, |x| < 1$

12. Estimate $\sin(100)$ using a third degree Taylor polynomial and the periodicity of $\sin(x)$. Use a calculating device to find the error in your estimate. Find the upper bound for the error using $|T_3(x) - \sin(x)| \leq \frac{k|x-a|^4}{4!}$ if $k \leq |f^{(4)}(x)|$ and compare it with your error.

13. Find the third degree Maclaurin polynomial for $f(x) = e^x \cos(x)$ by multiplying the third degree Maclaurin polynomials of e^x and $\cos(x)$ and discarding terms.

14. Find the fourth degree Maclaurin polynomial for $h(x) = \frac{1}{1+x}, |x| < 1$ by substituting $-x$ for x in the Maclaurin polynomial of $g(x) = \frac{1}{1-x}$.

15. Let $p(x) = \ln(1+x), |x| < 1$. The last exercise gives you the fourth degree Maclaurin polynomial for dp/dx. Integrate this polynomial to find the fifth degree Maclaurin polynomial for $p(x)$. How did you find the value of the constant of integration?

Find the Maclauren series for each function and its domain.

16. $f(x) = \frac{1}{1+4x^2}$

17. $p(t) = e^{t-2}$

18. Find the degree four Maclauren polynomial for $f(x) = (1+x)^{1/4}$

19. Find the Taylor series for $g(x) = \sin(x)$ centered at $a = \pi/2$.

20. Use a Taylor polynomial to estimate the value of $\int_0^1 e^{-x^3}\,dx$ with an error smaller than 10^{-2}.

Hint: An alternating series $\sum_{n=0}^{\infty}(-1)^n a_n$ with $a_n \to 0$ monotonically from above can be estimated with error $\le |a_{k+1}|$ by the kth partial sum $\sum_{n=0}^{k}(-1)^n a_n$.

21. Express $\int_0^x \ln\left(1+t^2\right)\,dt$, $|x|<1$, as an infinite series.

22. Evaluate the following.

(a) $e^{\frac{\pi i}{6}}$ (b) $e^{\frac{-\pi i}{3}}$ (c) $e^{\ln(3)-\frac{\pi i}{4}}$ (d) $e^{\pi i t}$

23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of the commands and answers. Find the Taylor polynomial of degree 10 for $\sec(x)$ expanded about $a=0$. Then use it to estimate $\sec(0.3)$ and find the error for that estimate.

Answers

1. converges
2. diverges
3. IC $=(-3,3)$
4. IC $=[-3,3)$
5. IC $=[-3,3]$
6. IC $=[-\sqrt{2},\sqrt{2}]$
7. IC $=(-1,1]$
8. IC $=[-1,1)$
9. IC $=(6,8)$
10. IC $=(-\infty,\infty)$
11. a) $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ b) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ c) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ d) $\sum_{n=0}^{\infty}x^n$
12. If using $\sin(100-30\pi)$ then $\sin(100)\approx -25.969$, error $=25.463$, within bound of 45.617;
If using $\sin(100-32\pi)$ then $\sin(100)\approx -0.506$, error $=0.00036$, within bound of 0.00331;
13. $T_3 = 1+x-\frac{x^3}{3}$
14. $T_4 = 1-x+x^2-x^3+x^4$
15. $T_5 = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}$
16. $\sum_{n=0}^{\infty}(-1)^n(4x^2)^n$, $|x|<1/2$
17. $e^{-2}\sum_{n=0}^{\infty}\frac{t^n}{n!}$, $|t|<\infty$
18. $1+\frac{x}{4}-\frac{3x^2}{32}+\frac{7x^3}{128}-\frac{77x^4}{2048}$
19. $\sum_{n=0}^{\infty}(-1)^n\frac{(x-\pi/2)^{2n}}{(2n)!}$
20. ≈ 0.81
21. $I=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n+1}}{n(2n+1)}$
22. a) $\frac{\sqrt{3}}{2}+\frac{i}{2}$ b) $\frac{1}{2}-\frac{i\sqrt{3}}{2}$ c) $\frac{3\sqrt{2}}{2}-\frac{3\sqrt{2}i}{2}$ d) $\cos(\pi t)+i\sin(\pi t)$