25 points   Math 160   Homework 8   Sections 12.1-12.5   23 problems

1. Find the $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ components of $\overrightarrow{PQ}$ if $P = (3,5)$ and $Q = (1,-4)$.

2. Calculate.

   (a) $< 2,1 > + < 3,4 >$

   (b) $5 < 6,2 >$

   (c) $(3\hat{\mathbf{i}} + \hat{\mathbf{j}}) - 6\hat{\mathbf{j}} + 2(\hat{\mathbf{j}} - 4\hat{\mathbf{i}})$

   (d) $\| < -6,14 > \|$

3. Draw $\vec{v} =< 2,3 >$ and $\vec{w} =< 4,1 >$ both with tail at the origin. Then sketch and label $2\vec{v}$, $-\vec{w}$, $\vec{w} + \vec{v}$, and $2\vec{v} - \vec{w}$.

4. $A = (2,2)$, $B = (-6,3)$, $P = (9,5)$, and $Q = (17,4)$. Are $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ equivalent? Are they parallel? Do they point in the same direction?

5. Find the given vector.
   (a) The unit vector $\vec{e_v}$ if $\vec{v} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$.
   (b) The vector $\vec{w}$ of length two opposite to $\vec{v} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$.
   (c) The unit vector $\vec{e}$ making an angle of $\pi/12$ with the x-axis.

6. Find all scalars λ so that $\lambda(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$ has length one.

7. Express $\vec{u} =< 3,-1 >$ as a linear combination $\vec{u} = r\vec{v} + s\vec{w}$ if $\vec{v} =< 2,1 >$ and $\vec{w} =< 1,3 >$. Then sketch $\vec{u}$, $\vec{v}$, $\vec{w}$ and the parallelogram formed by $r\vec{v}$ and $s\vec{w}$.

8. Find a position function for the line that passes through the two points $(1,2,3)$ and $(4,5,7)$.

Find the sum, dot product, and cross product for each pair of vectors in problems #9 to #12.

9. $\langle 1,1,1\rangle$, $\langle -3,1,2\rangle$

10. $\langle 5,-1,3\rangle$, $\langle -10,2,-6\rangle$

11. $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$, $\langle 2,1,-2\rangle$

12. $\hat{\mathbf{i}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

13. Which of the pairs of vectors in #9 to #12 are parallel vectors? Which pairs are perpendicular vectors?

14. What is the work done by a constant force represented by the first vector on a particle with displacement equal to the second vector for #'s 9 to 12? Assume the unit is J = Joules.

15. What is the area of the parallelogram determined by $\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$ and $\langle 2,1,-1\rangle$? What is the area of the triangle determined by these same two vectors?

Let $\vec{v} = \langle -1, 2, -2\rangle$ and $\vec{w} = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ for # 16 - 20.

16. Find $\|\vec{v}\|$, $\|\vec{w}\|$ and $\|\vec{v}+\vec{w}\|$. Verify that the sum of any two of these is larger than the third. This is the triangle inequality.

17. Find $\vec{v}_{\vec{w}\|}$, $\vec{v}_{\vec{w}\perp}$, $\vec{w}_{\vec{v}\|}$, and $\vec{w}_{\vec{v}\perp}$.

18. Find the cosine of the angle between $\vec{v}$ and $\vec{w}$.

19. Find the equation of the plane through the origin that is parallel to $\vec{v}$ and $\vec{w}$.

20. Find the equation of the plane through the point $(3, -1, 2)$ that is parallel to $\vec{v}$ and $\vec{w}$.

Let P be the plane $x + 4y + 7z = 3$ for #21 and #22.

21. Find two vectors that are parallel to P, but not parallel to each other.

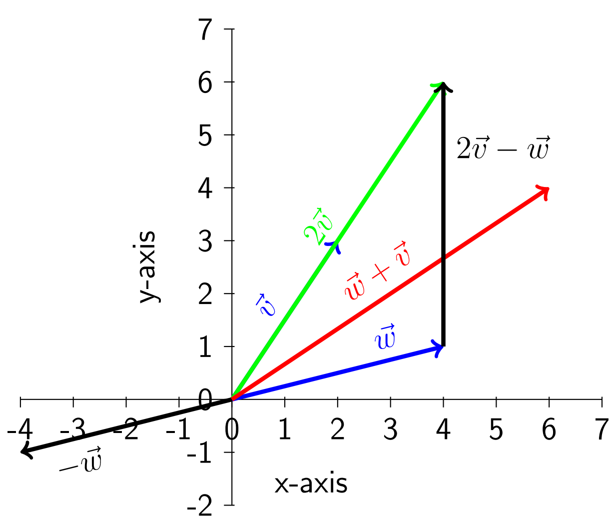
22. Find the equation of the plane parallel to P that passes through the point $(2, 1, 2)$.

23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of the commands and answers.

    Let $\vec{a} = \langle 4, 7, -1\rangle$ and $\vec{b} = \langle 3, 8, -2\rangle$. Use your CAS to find $\vec{a}_{\vec{b}\|}$ and $\vec{a}_{\vec{b}\perp}$.

Brief Answers

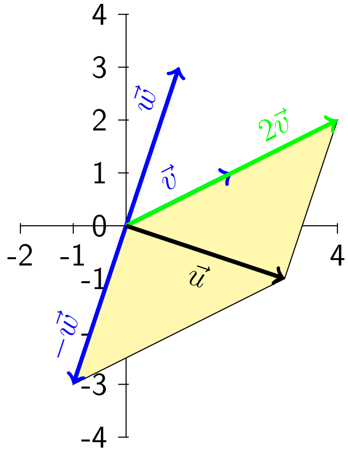
1. $\hat{\mathbf{i}}$-component $= -2$, $\hat{\mathbf{j}}$-component $= -9$
2. a) $< 5, 5 >$ b) $< 30, 10 >$ c) $< -5, -3 >$ d) $2\sqrt{58}$
3.



4. $\overrightarrow{AB} = -8\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\overrightarrow{PQ} = 8\hat{\mathbf{i}} - \hat{\mathbf{j}}$ are not equivalent, are parallel, but are not in the same direction.
5. a) $\vec{e_v} = \dfrac{< 3, 4 >}{5}$ b) $\vec{w} = \dfrac{2 < -1, 1 >}{\sqrt{2}}$ c) $\vec{e} = \dfrac{1}{2}\left\langle \sqrt{2+\sqrt{3}}, \sqrt{2-\sqrt{3}} \right\rangle$

6. $\lambda = \pm\frac{1}{\sqrt{13}}$

7. $\vec{u} = 2\vec{v} - \vec{w}$



8. $\vec{\alpha}(t) = \langle 1+3t, 2+3t, 3+4t\rangle$

9. sum $= \langle -2, 2, 3\rangle$, dot product $= 0$, cross product $= \langle 1, -5, 4\rangle$

10. sum $= \langle -5, 1, -3\rangle$, dot product = -70, cross product $= \langle 0, 0, 0\rangle$

11. sum $= \langle 3, 3, 0\rangle$, dot product $= 0$, cross product $= \langle -6, 6, -3\rangle$

12. sum $= \langle 2, 1, 1\rangle$, dot product $= 1$, cross product $= \langle -1, 1, 1\rangle$

13. The pairs in #'s 9 and 11 are perpendicular. The pair in # 10 is parallel. How do you know?

14. Work: 0 J, - 70 J, 0 J, 1J

15. $\sqrt{110}$ and $\frac{\sqrt{110}}{2}$

16. 3, $5\sqrt{2}$, $\sqrt{17}$

17. $\vec{v}_{\vec{w}\|} = -0.42\vec{w}$, $\vec{v}_{\vec{w}\perp} = \langle 0.26, -0.1, -0.32\rangle$, $\vec{w}_{\vec{v}\|}\frac{-7}{3}\vec{v}$, and $\vec{w}_{\vec{v}\perp} = \frac{\langle 2, -1, -2\rangle}{3}$

18. $\frac{-7}{5\sqrt{2}}$

19. $2x + 2y + z = 0$

20. $2x + 2y + z = 6$

21. Any two nonparallel vectors perpendicular to $\langle 1, 4, 7\rangle$. For instance, $\langle 7, 0, -1\rangle$ and $\langle 4, -1, 0\rangle$. Can you find a different pair?

22. $x + 4y + 7z = 20$