25 points   Math 160   Homework 9   Chapter 11   23 problems

1. **Sketch the trace of the parametric equations. The trace includes an arrow in the direction of increasing parameter.**

   (a) $x = 3 - 4t, y = 2 - 3t$
   (b) $x = \sin(t), y = \csc(t);\ 0 < t \leq \pi/2$
   (c) $x = 3 + 2\cos(2t), y = 1 + 2\sin(2t);\ \pi/4 \leq t \leq 3\pi/4$
   (d) $x = 5\sin(t), y = 2\cos(t);\ -\pi \leq t \leq 5\pi$

2. Find equations of tangent lines to the curve parameterized by $x = 2t^3$, $y = 3t^2 + 1$ whenever the slope of the tangent line equals one.

3. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for $x = 2\cos(t)$, $y = 3\sin(t)$; $0 < t < 2\pi$. At which of these points is the curve concave up?

4. Graph the trace of $x = e^{-t/2\pi}\cos(t)$, $y = e^{-t/2\pi}\sin(t)$, $0 \leq t < 2\pi$ and find its length.

5. Find the surface area obtained by rotating the curve of $x = \cos^3(\theta)$, $y = \sin^3(\theta)$, $0 \leq \theta \leq \pi/2$ about the x-axis.

6. Find the area enclosed by the x-axis and the curve of $x = 1 + e^t$, $y = t - t^2$.

7. Convert the polar coordinates to cartesian coordinates.

   (a) $(2, \pi/3)_P$
   (b) $(1, -3\pi/4)_P$
   (c) $(-1, \pi/2)_P$
   (d) $(1, \pi)_P$
   (e) $(2, -5\pi/6)_P$
   (f) $(-2, 3\pi/4)_P$

8. Convert the cartesian coordinates to polar coordinates.

   (a) $(2, -2)$
   (b) $(-1, \sqrt{3})$
   (c) $(3\sqrt{3}, 3)$
   (d) $(1, -2)$

9. Sketch the region in the xy-plane consisting of points with polar coordinates $2 < r < 3$, $5\pi/3 \leq \theta \leq 7\pi/3$.

Sketch the following polar relations in the xy-plane.

10. $r = 2\cos(\theta)$

11. $r = \theta$, $\theta \leq 0$.

12. $r = 1 - 2\sin(\theta)$

Convert to a polar equation in $r = f(\theta)$ form.

13. $y = 2$

14. $x + y = 2$

15. $x^2 + y^2 = 6x$

16. Find the points on $r = 1 + \cos(\theta)$ in the xy-plane where the tangent line is horizontal and vertical.

17. Find the exact length of the polar curve $r = e^{2\theta}$ for $0 \leq \theta \leq 2\pi$.

18. Find the area inside $r = 4 + 3\sin(\theta)$ for which $x \geq 0$.

19. Find the area enclosed by one loop of $r = \sin(4\theta)$.

20. Find the area that lies inside both $r = \sqrt{3}\cos(\theta)$ and $r = \sin(\theta)$.

Write a polar equation of a conic with focus at the origin and the given data.

21. Ellipse, eccentricity 1/2, directrix $x = 4$.

Identify the conic, its eccentricity, and directrix.

22. $r = \dfrac{3}{4 - 8\cos(\theta)}$.

23. CAS Problem (3 points): Use a CAS to solve the following. Submit a printed copy of your commmands and answers. Use an "ezpolar" command to graph $r = 4\sin(\theta) - 2$.

Answers

1. (a) Line through $(3, 2)$ and $(-1, -1)$.
   (b) Part of the curve $y = x^{-1}$ in the first quadrant starting near $x = 0$ and ending at $(1, 1)$.
   (c) From $(3, 3)$ to $(3, -1)$ on the left half of the circle with center at $(3, 1)$ and radius 2.
   (d) Start and end at $(0, 2)$ and wrap around the ellipse $4x^2 + 25y^2 = 100$ three times clockwise.

2. $y = x + 2$

3. $\dfrac{dy}{dx} = \dfrac{3\cos(t)}{-2\sin(t)}$; $\dfrac{d^2y}{dx^2} = \dfrac{-3}{4\sin^3(t)}$; concave up if $\pi < t < 2\pi$.

4. Curve is a counterclockwise spiral; arc length is $\sqrt{4\pi^2+1}(1 - e^{-1})$

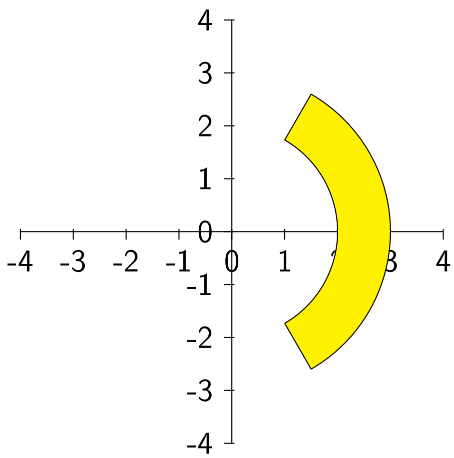
5. $\dfrac{6\pi}{5}$

6. $3 - e$

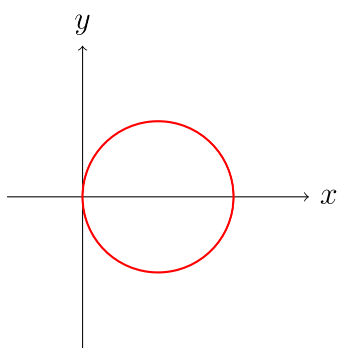
7. a) $(1, \sqrt{3})$ b) $(-\sqrt{2}/2, -\sqrt{2}/2)$ c) $(0, -1)$ d) $(-1, 0)$ e) $(-\sqrt{3}, -1)$ f) $(\sqrt{2}, -\sqrt{2})$

8. a) $(2\sqrt{2}, -\pi/4)_P$ b) $(2, 2\pi/3)_P$ c) $(6, \pi/6)_P$ d) $(\sqrt{5}, -\tan^{-1}(2))_P$

9.



10.



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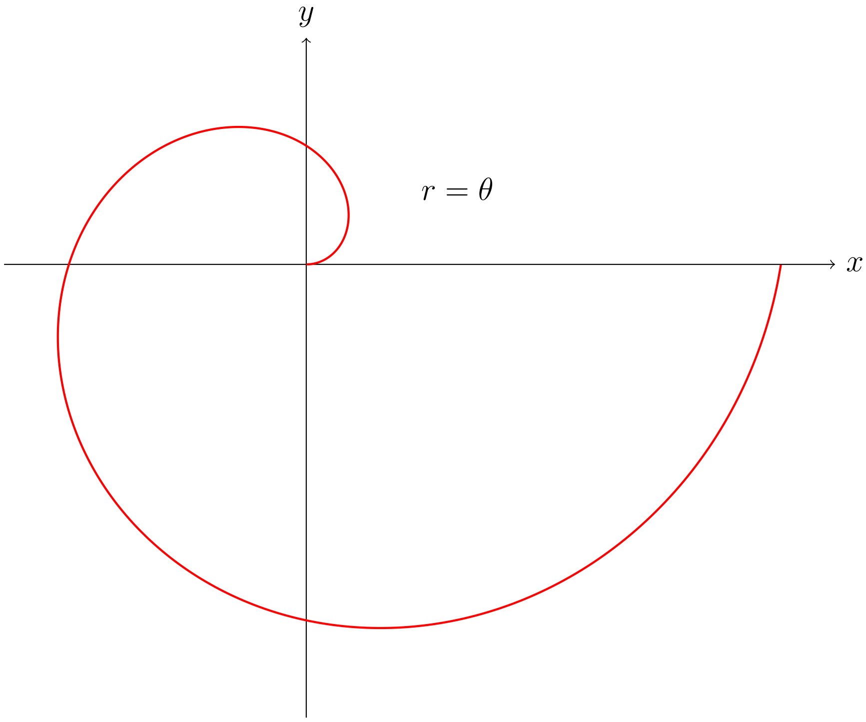

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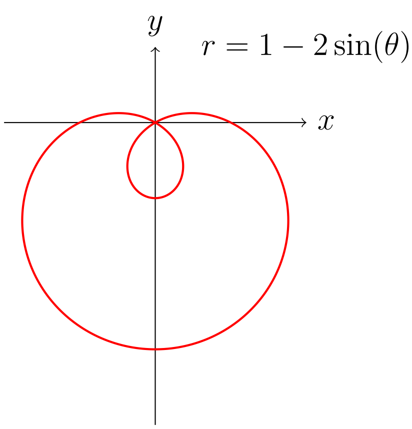

13. $r = 2\csc(\theta)$

14. $r = \sqrt{2}\sec(\theta - \pi/4)$

15. $r = 6\cos(\theta)$

16. horizonal at $(3/2, \pm\pi/3)_P$ and vertical at $(2, 0)_P, (1/2, 2\pi/3)_P, (1/2, 4\pi/3)_P$. No tangent line exists at $\theta = \pi$ since the tangent vector is $< 0, 0 >$.

17. $\dfrac{\sqrt{5}}{2}\left(e^{4\pi} - 1\right)$

18. $41\pi/4$

19. $\pi/16$

20. $\dfrac{5\pi}{24} - \dfrac{\sqrt{3}}{4}$

21. $r = \dfrac{4}{2 + \cos(\theta)}$

22. $e = 2$, hyperbola, $x = -3/8$