

Review of Integration by Parts (HW #2)

Integration by parts is the reverse of the product rule. If $u = u(x)$ and $v = v(x)$, then

$$uv' + u'v = (uv)'$$

$$\implies uv' = (uv)' - u'v$$

$$\implies \int uv' dx = \int (uv)' - u'v dx$$

$$\implies \boxed{\int u dv = uv - \int v du} \quad (*)$$

(*) is an identity and should be memorized if you can't derive it yourself from the product rule. There is a nice way to memorize all of the "parts" of this formula: it is called the tabular method.

$$\begin{array}{c|c|c} + & u & dv \\ \hline - & du & v \end{array}$$

Calculate $I = \int_0^{\pi/2} x \cos(x) dx$.

The tabular method really helps when integration-by-parts has to be repeated.

Calculate $I = \int_0^1 x^3 e^x dx$.

Find $I = \int_1^e \frac{\ln(x)}{x^2} dx$.

Calculate $I = \int_{-1}^1 \frac{\sin(x)}{1+x^2} dx$.

Calculate $I = \int_0^1 \sin^{-1}(x) dx$.

Calculate $I = \int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \sin(\theta^2) d\theta$.

Here is a special example where you must stop using parts and solve an algebraic equation.

Calculate $I = \int e^x \sin(x) dx$.

Find $I = \int_0^1 \frac{x^3}{\sqrt{9+x^2}} dx$.

Calculate $I = \int_0^1 \arctan(y) dy$.

As time allows:

(a) What is the derivative of $\sinh^{-1}(x)$ with respect to x ?

(b) What is the antiderivative of $\sinh^{-1}(x)$?