Trigonometric Integrals (HW #2)

The trig integrals you will see most often in future math courses are the ones used to develop the theory of Fourier Series. I call these the "Fourier Trig Integrals." The limits of integration are usually $-\pi$ to π, or 0 to 2π. One of the following is easy to calculate, while the others need trig identities. You can memorize all of these identities or derive them from three important ones:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \text{(Pythagorean Identity)}$$

$$\sin(a+b) = \sin(a)\cos(b) + \sin(b)\cos(a) \qquad \text{(Sine angle sum)}$$

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \qquad \text{(Cosine angle sum)}$$

We've already derived

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

and from this we can derive

$$\sin^2(x) = \frac{1-\cos(2x)}{2}$$

How?

We can use the identities to evaluate the following Fourier integrals:

$$\int_{-\pi}^{\pi} \cos^2(5x)\,dx$$

$$\int \sin^2(4x)\,dx$$

$$\int_{-\pi}^{\pi} \sin(4x)\cos(6x)\,dx$$

But what if the interval of integration is not symmetric about zero? We would need a third trig identity.

$$\int_0^{\pi/2} \sin(8x)\cos(3x)\,dx.$$

A similar identity works for the next two Fourier Integrals.

$$\int_{-\pi}^{\pi} \cos(5x)\cos(6x)\,dx$$

$$\int_{-\pi}^{\pi} \sin(9x)\sin(2x)\,dx$$

Here are some other trig integrals you should know how to integrate.

$$\int \tan(x)\, dx$$

$$\int \sec(x)\, dx$$

The Pythagorean identity helps us integrate $\int \sin^n(x)\cos^m(x)\,dx$.

$$\int \sin^3(x)\cos^2(x)\,dx$$

$$\int \cos^5(x)\,dx$$

We need another trig identity if the exponents are both even:

$$\sin(2x) = 2\sin(x)\cos(x)$$

since $\sin(x + x) = \sin(x)\cos(x) + \sin(x)\cos(x)$.

$\int \sin^2(x)\cos^4(x)\,dx$

Dividing the pythagorean identity by $\cos^2(x)$ gives $1 + \tan^2(x) = \sec^2(x)$.

$\int \tan^4(x)\sec^6(x)\,dx$ Hint: $\sec^2(x)$ is the derivative of what function?