Hyperbolics (HW #3)

Here is a review of hyperbolic functions. e^x is the most famous and useful function in mathematics, and so the hyperbolic functions are important too since they are made from e^x.

Definitions

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{has derivative} \sinh(x).$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{has derivative} \cosh(x).$$

Notice that

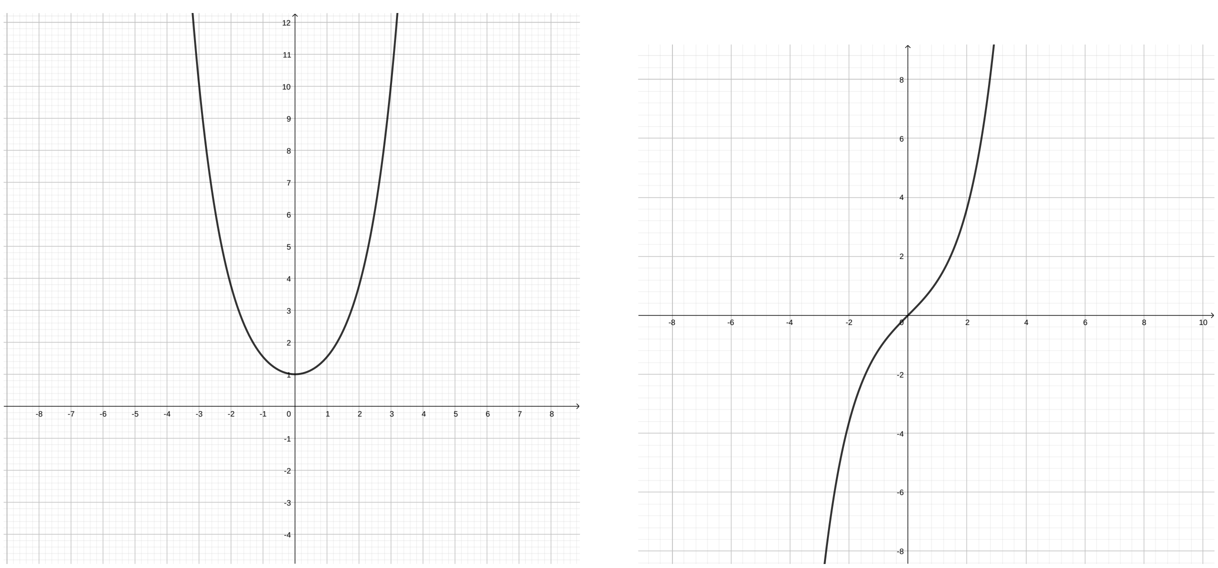
$$\cosh(x) + \sinh(x) = e^x$$

$$\cosh(-x) = \cosh(x)$$

$$\sinh(-x) = -\sinh(x)$$

show that $\cosh(x)$ is the even part of e^x and $\sinh(x)$ is its odd part.

Which of the following graphs represent $\cosh(x)$ and $\sinh(x)$?



Why "hyperbolic?" Because $x^2 - y^2 = 1$ is a hyperbola, and $\cosh^2(x) - \sinh^2(x) = 1$. You should prove that. Compare this to trig functions defined on the circle $x^2 + y^2 = 1$; in that case $\cos^2(x) + \sin^2(x) = 1$.

$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$ is defined analagous to $\tan(x)$, and the other hyperbolic functions are defined in a similar fashion.

Evaluate the following.

$$\int \tanh(x)\,dx$$

$$\int \cosh^2(x)\,dx$$

$$\int \sinh^3(x)\cosh^4(x)\,dx$$