Hyperbolics (HW #3)

Here is a review of hyperbolic functions. e^x is the most famous and useful function in mathematics, and so the hyperbolic functions are important too since they are made from e^x.

Definitions

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \quad \text{has derivative } \sinh(x).$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \text{has derivative } \cosh(x).$$

Notice that

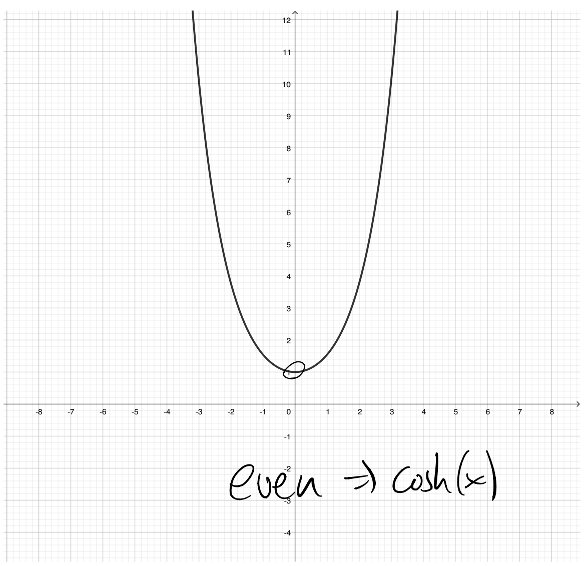
$$\cosh(x) + \sinh(x) = e^x$$

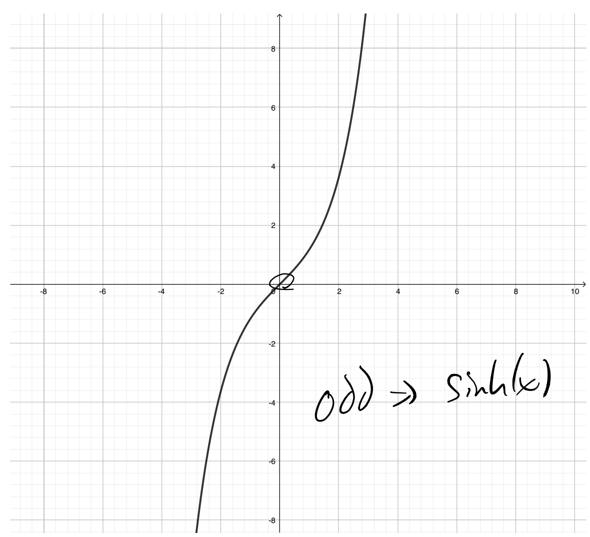
$$\Rightarrow \cosh(-x) = \cosh(x) \qquad \cosh(-x) = \frac{e^{-x} + e^{-(-x)}}{2} = \cosh(x)$$

$$\Rightarrow \sinh(-x) = -\sinh(x) \qquad \sinh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = -\sinh(-x)$$

show that $\cosh(x)$ is the even part of e^x and $\sinh(x)$ is its odd part.

Which of the following graphs represent $\cosh(x)$ and $\sinh(x)$?





Why "hyperbolic?" Because $x^2 - y^2 = 1$ is a hyperbola, and $\cosh^2(x) - \sinh^2(x) = 1$. You should prove that. Compare this to trig functions defined on the circle $x^2 + y^2 = 1$; in that case $\cos^2(x) + \sin^2(x) = 1$.

$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$ is defined analagous to $\tan(x)$, and the other hyperbolic functions are defined in a similar fashion.

Evaluate the following.

$$\int \tanh(x)\,dx = \int \frac{\sinh(x)}{\cosh(x)}\,dx = \int \frac{du}{u}$$

$u = \cosh(x)$
$du = \sinh(x)\,dx$

$$= \ln|u| + C$$

$$= \boxed{\ln(\cosh(x)) + C}$$

$$\int \cosh^2(x)\,dx = \int \left(\frac{e^x + e^{-x}}{2}\right)\left(\frac{e^x + e^{-x}}{2}\right) dx$$

$$= \int \frac{1}{4}\left(e^{2x} + 2 + e^{-2x}\right) dx$$

$$= \int \frac{1}{2} + \frac{1}{2}\cosh(2x)\,dx$$

$$= \boxed{\frac{x}{2} + \frac{1}{4}\sinh(2x) + C}$$

$(x^2 - y^2 = 1)$ $\quad \cosh^2(x) - \sinh^2(x) = 1 \Rightarrow \cosh^2(x) - 1 = \sinh^2(x)$

$$I = \int \sinh^3(x)\cosh^4(x)\,dx$$

$$= \int \sinh^2 \cosh^4(x) \cdot \sinh(x)\,dx$$

$u = \cosh(x)$
$du = \sinh(x)\,dx$

$$= \int (u^2 - 1) \cdot u^4\,du$$

$$= \int u^6 - u^4\,du$$

$$I = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$\boxed{I = \frac{\cosh^7(x)}{7} - \frac{\cosh^5(x)}{5} + C}$$