Probability and Improper Integrals (HW #3)

X = sum of the numbers on two rolled dice is a **Discrete Random Variable** because the possible outcomes of any roll is "countable" - in this case the outcomes are $2, 3, 4, ..., 12$.

A probability is a number between 0 and 1. The probability that $X = 7$ is written $P(X = 7)$, and

$$P(X = 7) = \frac{\text{samples of 7}}{\text{total samples}} = \frac{6}{36} = \frac{1}{6}.$$

What is the probability that $X = 5$?

B = the number of minutes waiting for a bus after the scheduled time of arrival is a **Continuous Random Variable**. In this case, $P(B = 2.1) = 0$, but $P(1 \leq B \leq 3) \neq 0$. B has a probability density function (pdf) associated with it, $f(x)$, which must have outputs that are nonegative and a total area under the curve equal to 1, and

$$P(1 \leq B \leq 3) = \int_1^3 f(x)\, dx.$$

Suppose $f(x) = \begin{cases} ke^{-0.5x} & x \geq 0 \\ 0 & x < 0 \end{cases}$. What must k be?

What is $P(1 \leq B \leq 3)$?

Improper Integrals

Any integral requiring a limit for its limits of integration is an **improper integral**. If the limit is a real number, then we say the improper integral **converges**. If the limit is infinite or does not exist, then we say the improper integral **diverges**.

Does $\displaystyle\int_1^\infty \frac{1}{x}\,dx$ converge or diverge? Defend your answer.

Does $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge or diverge if $p < 1$? Defend your answer.

Does $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$ converge or diverge if $p > 1$? Defend your answer.

For what values of p does $\displaystyle\int_0^1 \frac{1}{x^p}\,dx$ converge? Defend your answer. Hint: draw a picture and use symmetry about the line $y = x$.

Does $\displaystyle\int_5^\infty \frac{1}{\sqrt{x^4+1}}\,dx$ converge or diverge? Defend.

Definition: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx.$

Do either $\int_{-\infty}^{\infty} x\,dx$ or $\int_{-\infty}^{\infty} xe^{-x^2}\,dx$ converge? Defend your answers.

More practice as time allows.

Do the following coverge or diverge? Defend.

1) $\displaystyle\int_0^5 \frac{dx}{x^{19/20}}$

2) $\displaystyle\int_0^\infty te^{-st}\,dt, s > 0$ (this is a Laplace Transform used in math 220.)

3) $\displaystyle\int_1^2 \frac{dx}{\sqrt{x-1}}$

4) $\displaystyle\int_0^{\pi/2} \tan(x)\,dx$

5) $\int_1^2 \frac{dx}{x\ln(x)}$