Arc Length and Surface Area (HW #4)

Assume the x and y axes represent distance using the same unit of measure. If dx and dy are small, then define ds by

$$(ds)^2 = (dx)^2 + (dy)^2$$

Draw a picture to show that ds is a good estimate for the change in arc length, Δs, over a small interval.

Then the arc length of a graph of $y = f(x)$ from $x = a$ to $x = b$ is

$$\text{the arc length} = \sum \Delta s \approx \sum \sqrt{(dx)^2 + (dy)^2} = \sum \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \; dx$$

and this estimate becomes exact if we take a limit and obtain

$$\text{the arc length} = s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

or

$$\text{the arc length} = s = \int_a^b \sqrt{1 + (y'(x))^2} \, dx$$

What is the arc length of $y = \cosh(x)$ from $x = 0$ to $x = \ln(2)$?

What is the arc length of $y = \ln(\cos(x))$ from $x = 0$ to $x = \pi/3$?

Rotate the graph of $y = 2\sqrt{x}$, for $1 \leq x \leq 2$ about the x-axis to form a solid. What is the **surface area** of that solid?

How do we calculate the surface area of the solid formed by rotating $y = 2\sqrt{x}$, for $1 \leq x \leq 2$ about the y-axis?