Center of Mass (8.3) (HW #4)

The turning force (or torque) of a wrench is determined by the force applied to the end of the wrench and the length of the wrench.

If we place two kilograms on the x-axis 20 meters from the origin, for a moment we will have $2 \cdot 20 \cdot g$ units of torque about the y-axis. Put three more kilograms of mass at 50 meters from the origin. This gives us a total torque of $40g + 150g$ units. Where can I place the entire 5 kilograms of mass so that I have the same amount of torque?

Notice g is canceled out, so we define the center of mass of the masses without it:

$$\bar{x} = \frac{2 \cdot 20 + 3 \cdot 50}{5} = \frac{M_y}{M}$$

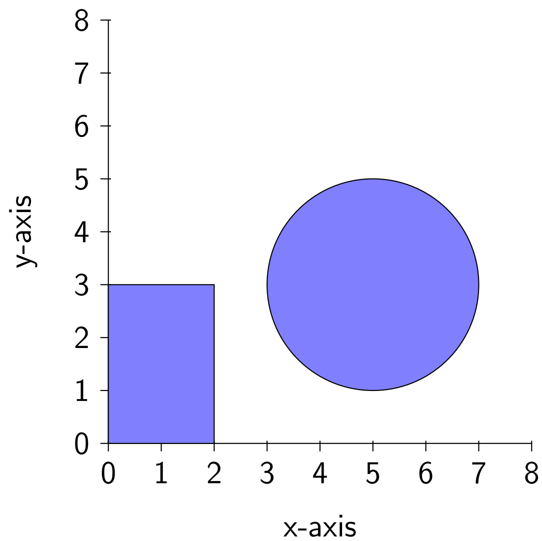
where M_y is the **moment about the y-axis** and M is the total mass.

Now suppose we have a thin horizontal wire with density $\delta(x) = x^2$ for $0 \leq x \leq 2$. Find its center of mass.

Now lets move to two dimensions. If we put two kilograms at the point $(20, 40)$ and three kilograms at the point $(50, 60)$ what is the center of mass of these masses? In this case,

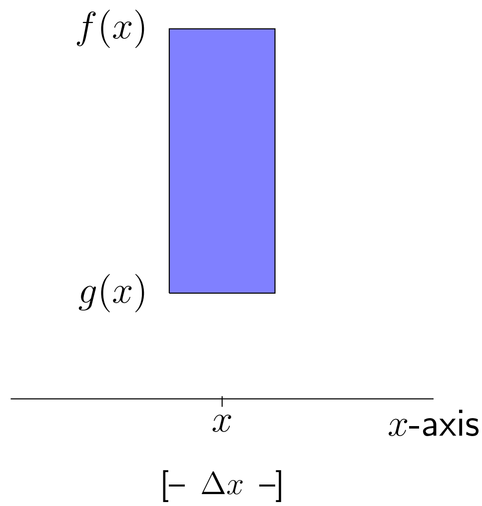
$$\bar{y} = \frac{M_x}{M}.$$

Now we suppose we have two dimensional "laminas" that have constant density k. Find the center of mass of the following lamina. Because the density of the lamina is constant, **the center of mass for any sub-lamina must lie on a line of symmetry**. We can then assume all the mass from each sub-lamina is placed at its center of mass and then proceed as we did in the last example.



The center of mass of a lamina with constant density is also called a **centroid**.

Find the centroid of the following lamina in terms of the midpoint of its width x and the upper and lower heights $f(x)$ and $g(x)$. Δx is the width of the rectangle.



Symmetry allows us to find the center of mass of each rectangle in a partition in just this way; we then assume all the mass of a rectangle is at that center of mass and have a finite number of weights as in our earlier examples. We apply this approach to find the centroid for a lamina bounded by curves.

Find the centroid for the lamina bounded by $y = x + 2$, $y = x^2$ and $x = 1$.

Why would the last problem be easier if the lamina were bounded by $x = 1$ and $x = y^2$?