Introduction to Differential Equations (HW #5)

$x + f(x) = x^2$ is an algebraic equation that can be solved for $f(x)$.

$\frac{dy}{dx} = 3x^2$ is a first order differential equation that can be solved for the function $y(x)$. Find its **general** solution. What would the **particular** solution be if the initial value $y(2) = 10$ were also to be satisfied?

An **nth order differential equation** is an equation with derivatives, the largest order of derivative being n. The **general solution** is a family of all the functions that satisfy the equation. If the differential equation is also given with an initial value, the solution is called a **particular solution**. The focus in this course will only be on first order differential equations.

A useful differential equation is $\frac{dy}{dx} = 2y$ if $y(0) = 3$. The best way to find the particular solution is to **guess and check**[1]. Do so.

[1]This is a valid method because there is a famous theorem called the Existence-Uniqueness Theorem that tells us there is a unique solution for every initial condition if the equation is nice enough. So if you can guess a solution for an initial condition, you have the right one.

Find the general solution for $\frac{dy}{dx} = 2y$ by guessing and checking.

This simple equation is incredibly useful. Here are some applications.

1) The Growth/Decay model. If P is the population at time t, then the Malthusian population model says that $\frac{dP}{dt} = kP$ where k is the growth constant. If we replace P with A that represents the amount of a radioactive substance, then we get the decay model. For instance, find the number of mold molecules, $P(t)$ on a piece of bread after 240 minutes (6 hours) if $P(0) = 2$ and $P(20) = 4$.

2) Newton's law Of Cooling. $T(t)$ is the temperature of an object at time t minutes, and T_s is the constant surrounding temperature. Then $\frac{dT}{dt} = k(T - T_s)$. A horse dies of colic and is found in his stall in the morning. At 6 a.m. the tempurature of the dead horse is $T(0) = 30°\text{C}$. At 7 a.m. $T(1) = 28°\text{C}$. The stall is kept at a constant temperature of 20°C. The normal temperature of a living horse is 38°C. What time did the horse die?

3) Free Fall.

$$ma = F = -mg - kv \implies v' = -g - \frac{k}{m}v$$

$$\implies \frac{dv}{dt} = -\frac{k}{m}\left(v + \frac{gm}{k}\right).$$

Solve this equation assuming $v(0) = 0$ and determine from that solution the terminal velocity.

4) Net flow = (flow in) - (flow out). You need to pay back a \$50,000 loan. Assume you pay it back continuously at \$500 per month = \$6000 per year. If the interest rate of the loan is 5% per year compounded continuosly, then how many years will it take to pay off the loan? So if $A(t)$ is the amount of dollars left to pay off at time t in years, then

$$\frac{dA}{dt} = 0.05A - 6000$$