Separation of Variables (HW #5)

Recall the model for population we used last lecture. The model is a good one until the mold has eaten the bread. At that time we would need a new model, the logistic model for population growth (section 9.4):

$$\frac{dP}{dt} = k(1 - \frac{P}{A})P.$$

where k and A are constants where it is understood that $A > P(t)$ is the carrying capacity.

The population growth constant has changed to a linear function of P. Guess and check will not help us now, so instead we recognize the function on the right is "separable." More generally, the equation

$$y' = f(x)g(y)$$

is separable because the function on the right can be factored into a function of the dependent variable times another function of the independent variable.

Solve $\frac{dy}{dx} = xy^2$ if $y(0) = 1$.

Solve $\frac{dy}{dt} = \frac{\sqrt{t}}{e^y}$.

Solve $\frac{dy}{d\theta} = \frac{e^y \sin^2(\theta)}{y \sec(\theta)}$.

Solve $\dfrac{dP}{dt} = \sqrt{Pt}$ for $P(1) = 2$.

Solve $\dfrac{dP}{dt} = k(1 - \dfrac{P}{A})P$

If $y(t)$ is the fraction of the population that has heard a rumor at time t in days, and $y(0) = 0.1$ and $y(2) = 0.4$, then we assume the equation is

$$y' = ky(1 - y).$$

In how many days will 75% of the population have heard the rumor? Notice that no more than 100% of the population can hear the rumor, so the carrying capacity has to be 1.