

Linear First Order Dif Eq (HW #5)

Section 9.5.

The standard form of a linear first order equation is

$$\frac{dy}{dx} + p(x)y = f(x) \quad (*)$$

Which of the following are linear first order equations?

1) $3y' + xy^2 = 1$ 2) $t \frac{dy}{dt} + t^2y = \sin(t)$ 3) $xy + \sqrt{x} = e^x \frac{dy}{dx}$

How do we solve them?

1) Put them in standard form (*). Go back to the linear equations above and put them into standard form.

2) Find an integrating factor $u(x)$ that completes the product rule on the left side of (*) if we multiply by it.

$$\frac{d(u \cdot y)}{dx} = u \cdot \frac{dy}{dx} + u \cdot p(x)y$$

Since $\frac{du}{dx} = u \cdot p(x)$ we must have

$$u(x) = e^{\int p(x)dx}.$$

Find the integrating factors for the linear equations above.

3) Multiply both sides of the equation by u , complete the product rule and write the left side as $\frac{d(u \cdot y)}{dx}$. Then integrate and solve for y .

Solve $y' + 3x^2y = 2xe^{-x^3}$ if $y(0) = 1$.

Solve $x \ln(x) \frac{dy}{dx} + y = xe^x$.

Example: Suppose a tank with capacity 500 liters starts with 200 liters of brine that has 20 grams of salt in it. Brine containing five grams of salt per liter enters the tank at three liters per minute, instantly mixes into a homogeneous brine within the tank, and exits the tank at two liters per minute. How much salt is in the tank at any time t ? What is the range for t ?