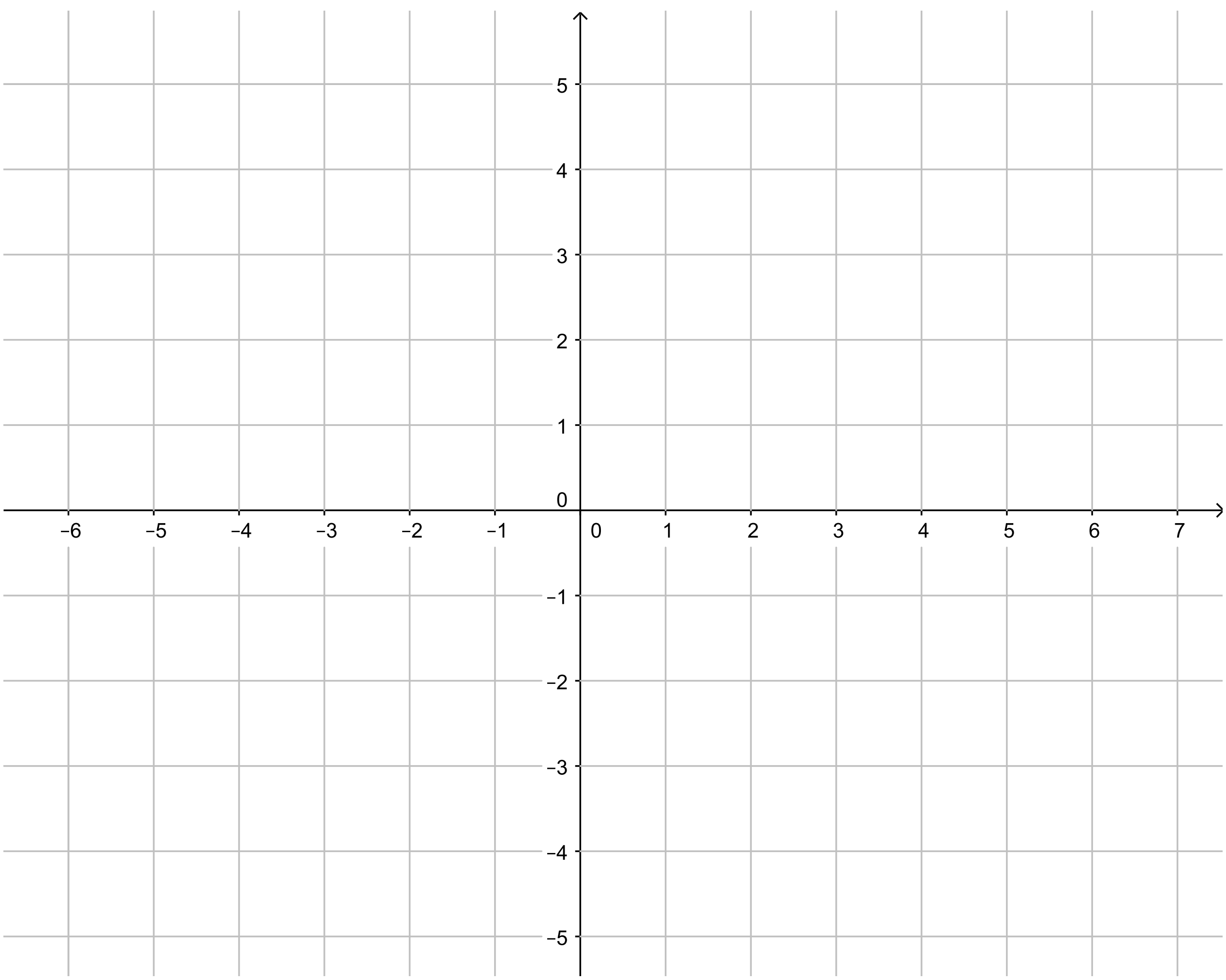
Euler's Method (HW #5)

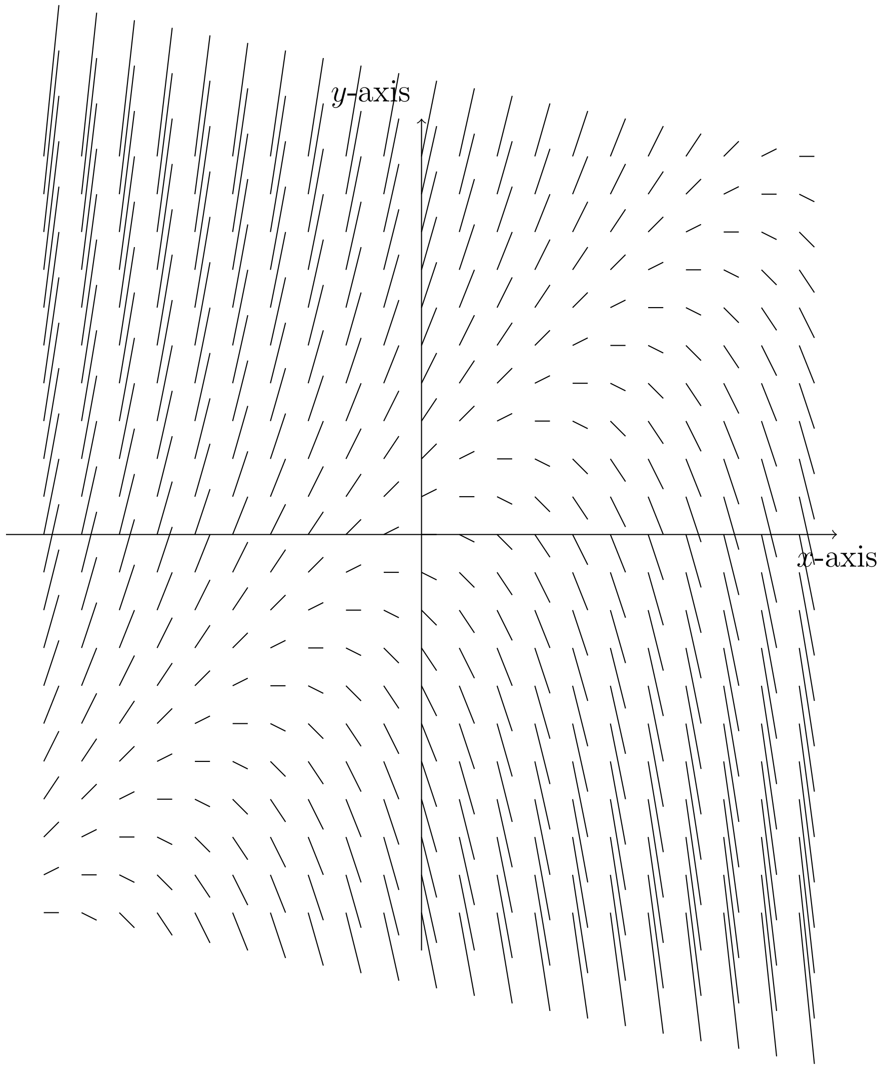
Section 9.3.

Usually we can only estimate solutions to differential equations using numerical methods. Most methods today such as Runge-Kutta are a bit too complicated for this class, but they all are similar to the first numerical method, Euler's Method, that is simple to explain. Euler's Method uses a slope field to estimate solutions.

Draw the slope field for $y' = y - x$. Draw several "isoclines" to help expedite your work.



Of course, no one draws slope fields by hand anymore unless they are students or teachers. My editing language, LaTex, allows me to easily draw slope fields. Here is $y' = y - x$. Sketch the solution if $y(0) = 0$. Now solve the IVP $y' = y - x$ if $y(0) = 0$ using our method for linear equations. Does your sketched solution approximate the exact one?



To sketch the estimated solution on the slope field, we follow a tangent segment until it ends and then start again using the next tangent segment closest to were we ended; then we repeat the process many times until we have a nice estimate. That is what Euler's Method does.

Here is the algebra. If the initial condition is $y(a) = b$, then we set $x_0 = a$ and $y_0 = b$. The tangent line through that point with slope $y'(x_0)$ is the Taylor polynomial $y = y_0 + y'(x_0)(x - x_0)$.

We choose a "step-size" h for our next input: $x_1 = x_0 + h$ and then use the linear approximation to define y_1: $y_1 = y_0 + y'(x_0)(x_1 - x_0)$. Repetition uses the recursive relation with $x_n = x_0 + nh$ below.

$$y_{n+1} = y_n + y'(x_n)(x_{n+1} - x_n)$$

We can also go backward if we allow n to be a negative integer.

I will not ask you to perform Euler's method; you should use a machine whenever it is used. However, I will ask you to identify which slope field corresponds to a given equation and to sketch particular solutions.

The slope fields for $y' = y - x^2$, $y' = y + x$, and $y' = y$ are given below. Write the differential equation under its corresponding slope field and defend your answers. Then sketch the particular solution for $y(0) = 0$ on each slope field using Euler's method.