Matlab for math 160 HW #4

Use the Simpson Method to approximate the integral of e^(x^2) from 0 to 1 with error less than 10^(-5). First decide on the even number of subintervals; that means we need to calculate the error and hence the max magnitude of the fourth derivative of e^(x^2) on the interval [0, 1].

```
syms x
D(x)=diff(exp(x^2),x,4)
```

D(x) = $12\,e^{x^2} + 48\,x^2\,e^{x^2} + 16\,x^4\,e^{x^2}$

On the interval [0, 1], D(x) is a maximum when x =1.

```
vpa(subs(D(x),x,1),6)
```

ans = 206.589

Use K = 207 in the error formula. Since (b-a)^5 = 1^5=1, the error 10^(-5) > Error implies

10^(-5)>207/(180n^4) so n is larger than

```
(207*10^5/180)^(0.25)
```

ans = 18.4151

Use n = 20 subintervals. I will create a numeric variable y which is an array of the endpoints of the subintervals. Matlab convention says the first one is y(1), not y(0), so start the for-loop at 1. The for-loop adds together the areas under the 10 parabolas.

```
y=0:1/20:1; I=0;
for i=1:2:19
    I=I+1/(3*20)*(exp(y(i)^2)+4*exp(y(i+1)^2)+exp(y(i+2)^2));
end
vpa(I,8)
```

ans = 1.4626536

Now use the "integral" command to check if our estimate is within 10^(-5) of the actual answer.

```
A = vpa(integral(@(p) exp(p.^2),0,1),8)
```

A = 1.4626517

```
abs(A-vpa(I,8))
```

ans = $0.0000018789791149664836211741203442216$

We indeed see our estimate is accurate enough.